A Method for Composing with Interval Cycles as Applied to an Original Composition for Symphony Orchestra

by

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ABSTRACT

Tonality is a method of composing with interval cycles that partition the octave. For centuries, this partition interval has been privileged for its acoustical properties. This dissertation takes a fresh look at partitions in general, and seeks to determine if there are broader principles capable of organizing interval cycles such as those used in tonality. However, it is deemed arbitrary to limit interval cycles to any one partition such as the octave. In this dissertation, interval cycles are demarcated into new hierarchical equivalence classes that cordon-off specific invariant traits related to how interval cycles interact in voice-leading space. These equivalence classes include prime-number class, partition class, cyclic family, and cyclic class. The multilevel coherence engendered by the method presented here is realized in the original composition *Symphony No. 1*.

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LIST OF ABBREVIATIONS

c	cardinality
cc	cyclic class
d	distance
lcm	least common multiple
p	partition
pnc	prime-number class
r	rotation
T	transposition
~	is equivalent to (equivalence relation)

CHAPTER I
INTRODUCTION

The purpose of this dissertation is to describe the creative process behind *Symphony No. 1*: an original composition for symphony orchestra. Table 1.1 shows a rough outline of this process. A series of experiments uncovered a set of integrated compositional methods and materials heretofore unknown. In the initial experiments, few creative choices were privileged or jettisoned.[1] Later, observed consistent outcomes emerged and supplied the groundwork upon which to establish steadfast rules, which were then chronicled, tested, and deliberately broken. The implementation and intuitive manipulation of these rules and procedures were at all times founded upon aesthetically sensitive aims. From this general creative process emerged a set of consistent procedures leading to a systematic method for composing with interval cycles.

Table 1.1. Steps Toward Creative Enlightenment.

Step 1.	Free Experimentation
Step 2.	Rule Formulation
Step 3.	Testing and Rule Breaking
Step 4.	Free Implementation

Carefully exploring the possibilities of rule infringement can help to delineate the boundaries, degree of flexibility, and applicability of any new compositional system. When or when not to break rules is beyond the ability of a mere computer program, since strict rule adherence is aesthetically ineffectual. Even programmed parameter randomization without human intervention would fail to capture the drama, timing, and spontaneity so essential to the symphonic narrative. This does not countermand the important status rightfully attributed to computer-aided composition.[2] In fact, *Symphony No. 1* was composed almost exclusively on the computer screen.[3] Yet it is incumbent upon the human mind to effectively implement a compositional method, especially when so many compositional choices are left unaffected by its range of parameters. For example, when should a construction be verticalized or linearized, or how long should a specific parameter remain invariant before a new one is added or takes over? The human composer can best answer these sorts of questions.

Reason dictates as inevitable the emergence of multitudinous post-tonal approaches given the undesirable strictures of tonality and the perceived inadequacies of competing methods in the achievement of individually relevant aesthetic aims. Such approaches continually engender new measures of coherence from whence a composition, movement, or phrase may be unified, while simultaneously enabling the availability of inexhaustible timbres, sonorities, and scale tunings from which a full spectrum of previously unattainable choices becomes unlocked. Yet every increase of choice must be tempered by an equal increase of control: there is a one-to-one correspondence. A system provides a means with which to narrow an infinite quantity of alternatives, acting as a sieve to bring the most salient choices into the foreground. Some of these sieves are entirely subjective, while others are of more practical use. Most sieves are explicit and measurable, while others are implicit, eluding quantification.

Although not fully explored here, the method developed for *Symphony No. 1* may be employed as an analytical tool for the examination of other post-tonal works. Indeed, it is hoped that it can serve concurrently as both a prescriptive and descriptive approach. It should be stressed that it is not an evaluative approach, since nonconformance is at all times encouraged: fulfillment of rules does not denote quality or indicate merit. Consciously or unconsciously followed rules simply imply that patterns are effective communicators of meaning and coherence.

[1] Exceptions include the usual suspects: tonal melodies and harmonies, which were immediately jettisoned, while whole-tone and octatonic constructions were used sparingly.

[2] To be distinguished from a composition written exclusively by a computer program.

[3] Vis-à-vis user manipulation of onscreen elements, as opposed to traditional pencil and staff paper methods.

This method deals primarily with quantifying the relationships integrating vertically-aligned interval cycles such that no pitch is extraneous. As such, cycles are akin to chords found in tonal music since every pitch and melodic contour exists as a result of a controlled harmonic progression. Within a succession of vertically aligned structures, pitch choice within the resultant horizontal line is often restricted by voice leading restrictions. Some of the contrapuntal restrictions employed in *Symphony No. 1* have been with us for centuries. For instance, voice crossing as defined here is virtually identical to that of the early contrapuntalists such as Giovanni Palestrina. As was surely known by these early masters, a careful selection of materials can minimize limitations that obstruct aesthetic intentions. Indeed, it is entirely possible to accommodate a preexisting melodic line, circumnavigating and even ignoring theoretical strictures. Moreover, other variables such as rhythm, texture, and dynamics remain quite independent of systemization.

Besides being intrinsically valuable in the aesthetically disinterested sense, an original method may serve as a framework for future compositions, thereupon attaining practical value. A composer must consistently devise tools and develop shortcuts to expedite the creative process. On that note, this document is primary research of the causal and experimental vein. It does not aspire to the inane task of ‘comparing and contrasting’ the findings and opinions of other authors, as we have seen in so many of the recent dissertations. Rather it aspires to answering questions that we do not yet no the answers to.

Yet, in articulating the newness of this particular method it is important not to overemphasize it by neglecting to locate it historically. Clearly, examples of interval cycles may be seen most strikingly in the music of Béla Bartók, Alban Berg, Charles Ives, Witold Lutosławski, and Igor Stranvinsky. Music theorists such as Elliot Antokoletz, Richard Cohn, Dave Headlam, Philip Lambert, and George Perle have presented very sophisticated explanations of interval cycles. This document can be distinguished from these writings primarily because my purpose here is to present a practical approach for incorporating interval cycles into ones composition rather than to analyze a preexisting composition.

It should be noted that many of the techniques employed in *Symphony No. 1* are based on certain rudimentary aspects of mathematics. These techniques are described with the non-mathematician in mind: definitions and prodigious examples are incorporated wherever practicable. Throughout this dissertation may be found several new terms heretofore absent from the music theory lexicon. These terms generalize critically important concepts, filling in the gaps, as it were, that once prevented the full understanding and operative implementation of interval cycles.

What follows can roughly be divided into two halves. The first half (Chapters II-IV) examines the method of composing with interval cycles, while the second half (Chapter V) deals with how the method is applied in *Symphony No. 1*. Specifically, Chapter II explores the general properties of interval cycles, Chapter III introduces a new approach to voice leading in vertically aligned interval cycles, Chapter IV examines the implications of composing with interval cycles as partitioned into equivalence classes, and Chapter V analyzes key moments of *Symphony No. 1*, which reveal the macro and micro forms in context of the compositional method explained in the first half. At the end of his book on Alban Berg, the author Dave Headlam writes that “What remains for the theory of symmetry is a systematic formalization of objects and transformations, consonance and dissonance, and controlling and controlled structures, comparable to that of set theory and Schenkerian theory” (Headlam 386). It is believed that this document brings us closer to such a theory.

CHAPTER II
INTERVAL CYCLES

Every vertical and/or horizontal structure in *Symphony No. 1* is derived from an interval cycle, an invariant feature providing overall cohesiveness. The term *interval cycle* describes an ordered pattern of intervals fixed in pitch space.[4] This pattern may occur once, or may be vertically duplicated (or stacked) as many times as desired. For instance, the interval cycle [1-2] generalizes all of the pitch-class sets (013), (0134), (01346), (013467), (0134679), and (0134679T). That is to say that the label [1-2] is used regardless of how many stackings of the cyclic pattern occur in the music. In other words, cycle [1-2] is equivalent to cycle [1-2-1-2] and cycle [1-2-1-2-1-2]. This generalization, which may be called *stratum equivalence*, is an important concept that will allow for further classifications later. At least two stackings are necessary for positive cycle identification.

Cycles are depicted as an ordered set of integers within brackets, where integers indicate a *pitch interval* measured in semitones between adjacent pitches.[5] These intervals are not to be confused with *interval classes* unless otherwise indicated.[6] The term 'cycle' encompasses single-interval cycles such as the circle of fifths [7] as well as multi-interval cycles, such as the octatonic scale [1-2].[7] The number of intervals a cycle has per single stacking is called its *cardinality*.[8] For example, the circle of fifths [7] has a cardinality of 1 (thus can be called a monad) regardless how many pitches or stackings are actually present.

Every interval cycle partitions a specific pitch interval. As defined in number theory, a partition of a number *n* is a way of writing *n* as a sum of other numbers. For example, the number 5 can be partitioned into six different interval cycles: [5], [1-4], [2-3], [1-1-3], [1-2-2], and [1-1-1-2]. Notice that [1-1-1-1-1] (the chromatic scale) is excluded from this list, since the *cycle normal form* (CNF) for the chromatic scale is simply [1].[9] Cycles are always written so that the smallest intervals are stacked to the left and the largest intervals are stacked to the right. This is, roughly, the normal form for cycles.[10] Figure 2.1 shows two stackings of the diatonic scale, cycle [1-2-2-1-2-2-2]. The diatonic scale is a partition of 12 since the intervals add up to 12.

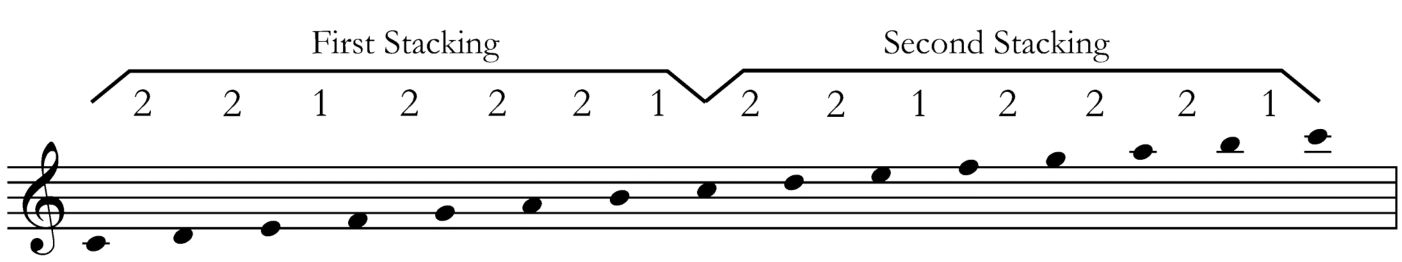


Figure 2.1. Two stackings of the diatonic cycle [1-2-2-1-2-2-2].

Interval cycles that share a common partition interval are placed into the same *partition class*. The diatonic scale is a member of partition class 12, as are the chords in the tonal chord progression shown below in Figure 2.2. The tonal cycles in Figure 2.2 are subsets of the cycle in Figure 2.1. This cycle-within-a-cycle technique is used extensively in *Symphony No. 1*. In every case the superset acts like a key signature: a referential construct from which all chords and pitches are derived. As mentioned earlier, at least two stackings of a cyclic pattern are sufficient for us to identify it with certainty. This may be called the *two-stacking rule*. If one stacking or less of a cyclic pattern is present, its identity may prove elusive. That's not to suggest that such a tactic is undesirable. The cycles in a typical harmonic progression often contain one or fewer stackings. If we are

[4] The term *interval cycle* is at times shortened to *cycle* throughout this document. Theorists such as Philip Lambert and Richard Cohn prefer to use the term *transposition cycle* rather than *interval cycle*.

[5] If a particular tuning system uses intervals smaller that the semitone, then the integers represent the smallest interval used.

[6] A pitch interval is the actual distance between two pitches measured in semitones, whereas an interval class is a reduction of this interval vis-à-vis inversional equivalency modulo 12. There are an infinite number of pitch intervals, but only seven interval classes: 0, 1, 2, 3, 4, 5, and 6.

[7] In many previous discussions of interval cycles, a single-interval approach has been adopted and extended to conceptualize all interval cycles.

[8] Using the term cardinality to count the number of intervals is not to be confused with its traditional usage to count the number of pitches (for example see Lambert 1997).

[9] CNF reduces a cyclic designation into single-stacking form as well interval normal form (INF).

[10] See John Clough's description of INF in his "Aspects of Diatonic Sets," 1979.

familiar with the conventions of tonality, we know that triads are partitions of the octave, rendering cyclic identification unproblematic.[11]

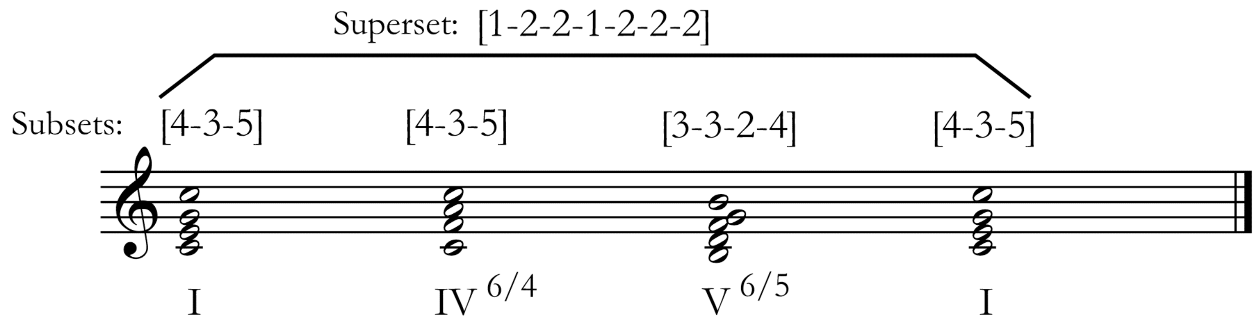


Figure 2.2. Cyclic subsets of cycle [1-2-2-1-2-2-2].

It should be obvious that the intervals of a cycle may be rotated without changing the identity of the cycle so long as their relative positions do not change. Thus, r*otation* does not change cyclic identity. For any interval cycle, the number of possible rotations is equal to that cycle's cardinality ($r = c$). As Figure 2.3 shows, the "major triad" has three possible rotations: [4-3-5], [3-5-4], and [5-4-3]. Rotation is analogous to the tonal concept of chordal inversion since, for example, C-E-G, G-C-E and A#-C#-F# are all "triads."

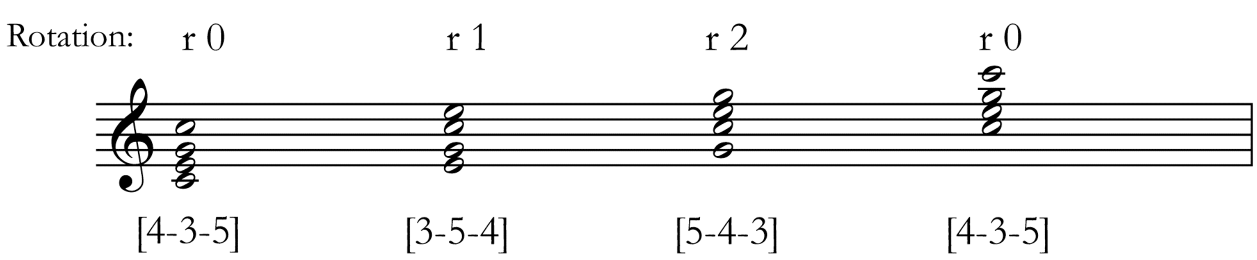


Figure 2.3. Rotation is equivalent to chordal inversion.

The diatonic scale is not the only cycle that divides a single octave. For example, cycles [2], [3], [4], and [6], also divide an octave into equal parts. The octave is divisible by their partition intervals even though these cycles are not partitions of 12 per se. For any single-octave cycle, the number of times 12 can be divided by its partition *p* is the number of times it may occur without duplicating pitch classes. This tells us when the cycle is complete. In complete cycles, the first pitch of the initial cycle is always identical to the first (or last) pitch of the terminal cycle. For instance, the diatonic cycle divides 12 once, so is complete after one stacking. Béla Bartók's Z cell [1-5] is a partition of 6, thus completion occurs after two stackings.

All of Olivier Messiaen's seven "modes of limited transposition" are further examples of octave-dividing partitions. For instance, Messiaen's mode III, interval cycle [1-1-2], is complete after three stackings (12 ÷ 4 = 3). Figure 2.4 clearly shows a complete [1-1-2] cycle, since the first pitch of the initial stacking (C) is identical with the last pitch of the terminal stacking (C). Note that the ending point of one stacking is always the starting point for the subsequent one. This makes counting pitches somewhat counterintuitive. For example, one stacking of [1-1-2] consists of four pitches (C, C#, D, E), while two stackings of [1-1-2] consist of only seven (C, C#, D, E, F, F#, G#). Since the ending of one stacking is the beginning of the next, the pitch E is only counted once. Thus, each stacking is interlocked with the next by means of a common axis or pivot point.

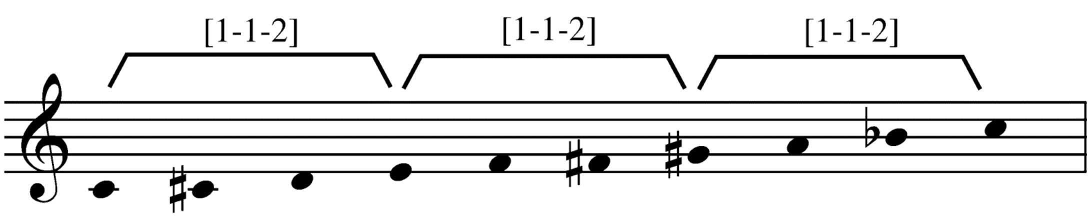


Figure 2.4. Messiaen's third mode of limited transposition.

[11] Cycle identification alone is not a substitute for Roman numeral analysis in tonal settings.

For all non-octave dividing interval cycles, *cyclic completion* can be described as follows: a complete interval cycle is one in which the first pitch of the initial stacking is identical with the first (or last) pitch of the terminal stacking. The number of stackings a cycle must traverse in order to achieve completion is called its *cyclic completion quotient*. This quotient is determined by finding the least common multiple (LCM) of a cycle's partition *p* and 12, and then dividing this number by *p*:

$$\frac{\text{lcm of p and 12}}{\text{p}} = \text{cyclic completion quotient}$$

For instance, the circle of fifths [7] has a partition of 7. The LCM of 7 and 12 is 84. Dividing 84 by 7 tells us that the circle of fifths must repeat 12 times in order to be complete. Mathematically, the circle of fifths is optimal. An *optimal interval cycle* is one with a cyclic completion quotient of 12. A *non-redundant* cycle is one that does not contain repeated pitch-class content upon cyclic completion. All single-octave cycles are non-redundant and, with the exception of the chromatic scale, are non-optimal. Non-redundant cycles are unique in their ability to maximize pitch-class variety.

A *twelve-tone cycle* will be defined here as one in which all twelve pitch-classes are stated *at least* once upon cyclic completion. This definition means that a twelve-tone cycle may have pitch-class repetition upon competition, thus need not be non-redundant. For this reason, a twelve-tone cycle is not to be confused with a twelve-tone row. Furthermore, a twelve-tone cycle need not necessarily be an optimal cycle, since it is certainly possible for twelve pitch classes to be stated within the span of fewer than 12 stackings of the cyclic pattern.

Figure 2.5 shows cycle [1-3-1-5], which is complete after only six repetitions. Note that every pitch class occurs exactly twice (at the pitch interval of 24). The composer should deliberately choose whether or not to allow pitch class repetition by carefully checking for multiples of 12 within a complete cycle. In a similar example, we can clearly see that while cycle [1-2-10] is a partition of 13 and is thus is an optimal cycle, it contains a pitch-class repetition at the pitch interval of 12 (adding its last two intervals gives us 2 + 10 = 12).

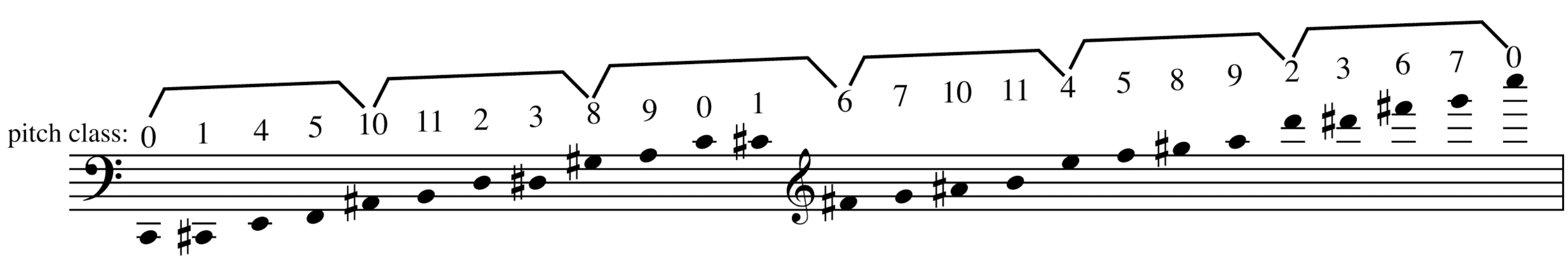


Figure 2.5. Cycle [1-3-1-5] is complete after fewer than twelve stackings yet traverses twelve pitch classes.

In *Symphony No. 1*, interval cycles that are any combination of optimal, non-redundant, and twelve-tone are used in much greater frequency but not necessarily to the total exclusion of than those that are not. This reflects the composers desire to confine the compositional materials to those that maximize uniqueness and variety. These classifications represent important measures of compositional usefulness and exclusivity. In cases where interval cycles are not optimal, non-redundant, or twelve-tone, certain strategies can be adopted to mitigate these shortcomings, thus rendering them imperceptible. For instance, if only two stackings of a [1-3-1-5] cycle are used, its pitch class repetition will be concealed.

Single-octave partitions take advantage of acoustical realities such as octave equivalency, pitch centricity, and the harmonic series. Although it is understandable why the octave partition has played such a large role in music of the past, it is by no means the only partition available. In my view, the octave has been given undue privilege and is frankly overused. For that reason, single-octave divisions are avoided in *Symphony No. 1*. It should be noted that Alfred Cramer (2003) has made some very interesting observations about how and why certain "altered octaves" can fuse and take on a perceptual coherence sufficient enough to supplant traditional harmonies. Indeed, Cramer claims that certain altered octaves (specifically those separated by 11 or 13 semitones) are essential to Schoenberg's concept of the term *Klangfarbenmelodie*. Cramer points out that critical bandwidth masking is most pronounced in partitions of 11 and 13. These intervals are used heavily in *Symphony No. 1*, and at the same time, perceptual grouping is reinforces the partition interval (whatever it might be) by means of parallel

motion (or simultaneous onset of tones). As Cramer points out, parallel motion has been well documented to contribute to a strong sense of fusion.

Even if a complete cycle is non-redundant and twelve-tone, it may simply prove impractical to use in its complete form due to range limitations. And yet, these limitations are easily overcome. By transposing subsets of a large multi-octave cycle, one can eliminate range issues while maintaining pitch class variety. Figure 2.7 shows incomplete [7] subsets gradually unfolding a complete twelve-tone aggregate vis-à-vis transposition. Likewise, a cycle that is not twelve-tone may yet unfold a twelve-tone aggregate using transposition. For example, the complete whole-tone cycle [2] need only be transposed by one semitone to generate all twelve-tones.[12]

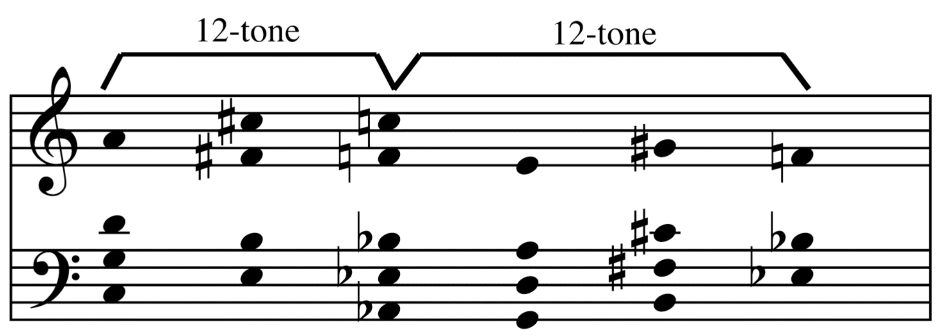


Figure 2.7. Transpositional completion of twelve-tone aggregate using transposed [7] cycles.

Since there are an infinite number of interval cycles, it is necessary to bring the most relevant choices into the fore. Without going into how cycles can be grouped, there are a number of ways to immediately filter out cycles that may be considered to be less exclusive. First, cycles must obviously fall within naturally occurring acoustical boundaries, such as the range of human hearing (20 to 20,000 Hz), within the range of the chosen instrumental ensemble, above the critical bandwidth (low range), and below the “shrillness” range.

The second consideration has to do with filtering out undesirable sums and sets, especially those that would be suggestive of tonality, such as the octave or the tritone. In twelve-tone tuning, cycles that contain an embedded sum 12 (or some multiple of 12) should be filtered, such as [6], [1-2], [1-11], [1-2-10], or [2-5-1-10], or simply all multiples of 6. Similarly, certain embedded subsets are also avoided. Cycles containing the succession of intervals 3-4 or 4-3 (which strongly suggest minor and major triads) are ardently avoided, for example cycles like [3-2-4] and [1-3-4-7]. Finally, subsets such as 1-1-1, 2-2-2 or 3-3-3 are used sparingly.

Cycles may also be limited to the palindromic (or non-invertible) type. As palindromic cycles are the same forward or backward, inverting or retrograding them does not change their identity. Some palindromic cycles include:

1. Any one-interval cycle, such as [7].
2. Any two-interval cycle, such as [1-3]
3. Any three-interval cycle with two of the same interval number, such as [1-1-3] or [2-2-5].
4. Cycles such as [2-2-2-5], [2-2-5-5], [1-5-2-5], and [1-1-5-3-5] to name a few.

Table 2.1 lists all of the palindromic cycles partitioning 13, without sum 12, and without the embedded subsets [3-4] and [2-2-2-2].

[12] Incidentally, this is why Messiaen calls cycle [2] “mode I.”

Table 2.1. Filtered Partitions of 13.

Cardinality:	Cycles:
1	[13]
2	[2-11] [3-10] [4-9] [5-8] [6-7]
3	[2-2-9] [3-3-7] [3-5-5] [4-4-5]
4	[2-3-2-6] [2-4-2-5] [2-3-5-3]
5	[2-2-3-3-3] [2-3-2-3-3]

CHAPTER III
VOICE LEADING AND VERTICALLY-ALIGNED CYCLES

Progressions of vertically-aligned cycles form the harmonic basis for *Symphony No. 1*. Just like progressions involving tertian chords, voice leading between a pair of cycles may involve any combination of common tone (sustained) motion, parallel motion, similar motion, and contrary motion. A simple transposition results if voices move in all-parallel motion, therefore, this is largely avoided. In order to ensure smooth voice leading, these progressions must consist of cycles that are related[13]. Two cycles are related when *voice conservation* occurs, meaning that both cycles have the same number of voices. The following examples will serve to illustrate this relation.

Progressions of related cycles will always contain *cyclically-adjacent voices*, which are voices locked in parallel motion by an interval that is equal to the *least common multiple*, or *LCM,* of the partitions of the cycles. In tonal music, cyclically adjacent voices always move in parallel octaves, a limitation that from the 21st century perspective, may seem somewhat arbitrary. Of course, this parallel motion is sometimes merely implied by the context, since it is only present when there are two or more stackings of a cycle.

Figure 3.1 shows two stackings of cycle [2-5] moving in similar motion to cycle [2-5]. Both cycles are partitions of 7, so the LCM is also 7. It follows that the cyclically adjacent voice pairs (C, G) and (D, A) move in parallel motion to (C#, G#) and (F#, C#) respectively.

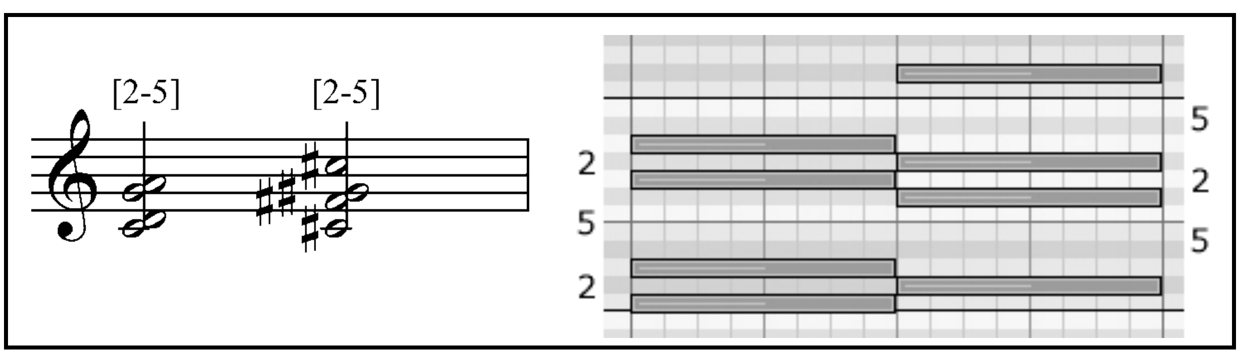


Figure 3.1. [2-5] moving in similar motion to [2-5].

The *voice-leading distance* between a pair of adjacent cycles in pitch space is the sum of the voice motion within a single LCM.[14] In Figure 3.1, the voice motion within the LCM of 7 is C +1 and D +4, thus their distance is 1 + 4 = 5. Note that the motion of voices beyond the first LCM (or stacking) is not counted when calculating voice-leading distance because of stratum equivalence. For this reason, the measure of distance defined here is quite distinct from more typical measures such as those discussed by Joseph Strauss.

Similar to tonal counterpoint, voice crossing is strictly avoided. The *voice-crossing rule* may be generalized as follows: when one voice X is moving toward another voice Y that is moving in the same direction or is sustained, the movement of X can be no greater than one less than the interval between X and Y. For instance, in Figure 3.1, the immediately adjacent interval between G and A is 2, so G may move a maximum of +1.

As a consequence of the voice-crossing rule, every cycle is limited to a *maximum allowable voice-leading distance.* This distance is an *absolute* distance, meaning that negative motion is counted as a positive motion. To determine this distance, simply subtract the partition interval p per LCM by the cardinality c per LCM: $p - c = d$ where d is an absolute (positive and/or negative) measure of motion in semitones. It is usually desirable for this distance to be greater than 2, an observation acknowledging the fact that cycles such as [1-1-1-2] are often better utilized as scales rather than as vertical constructions. For Figure 3.1, we get 7 - 2 = 5 as a maximum allowable distance, which may be thought of as a potential upper limit. In practice,

[13] By my definition "smooth voice leading" does not necessarily imply stepwise (or parsimonious) motion.
[14] See Joseph Strauss (2005) for a discussion of *voice-leading space* and voice-leading distance.

voice motion can be, and usually is, less than the maximum. In Figure 3.1, the *absolute* distance traversed is the maximum: |1| + |4| = 5. A graphical representation of pitch space is provided in this and other examples in order to make certain concepts more clear[15].

Figure 3.2 shows a common-tone progression from [2-5] to [1-6], with the cyclically adjacent pair (D, A) moving the maximum allowable distance (+4), while the other voices are sustained. The total voice-leading distance traveled per LCM (7) is 0 + 4 = 4, which is also the absolute movement (less than the maximum allowable distance of 5). Sustained pitches are vital to maintaining independent voices within a polyphonic texture. They are especially vital in progressions involving cycles, since they tend to lessen the audibility of the cyclically adjacent voices moving in parallel motion.

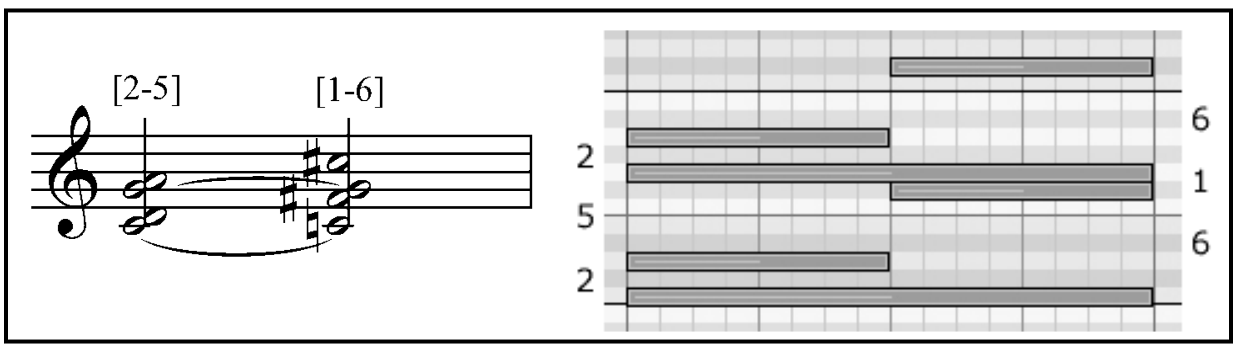


Figure 3.2. [2-5] moving with sustained motion to [1-6].

Figure 3.3 shows cycle [2-5] moving in contrary motion to [1-6]. The total voice-leading distance traveled per LCM is -3 + 1 = -2. The total *absolute* distance traveled is |-3| + |1| = 4 (less than the maximum allowable distance of 5). For contrary motion, the voice-crossing rule is expressed as follows: when two voices move in contrary motion toward each other, the *sum* of their absolute movements can be no greater than one less than the interval between them. In this case, the sum of the absolute movements, 4, is one less than the interval between them, 5. Note that in this case, it would be impossible for contrary motion to occur in the opposite direction without voice crossing (C and D moving toward each other) because their absolute movements cannot sum to 1 (one less than the adjacent interval 2).

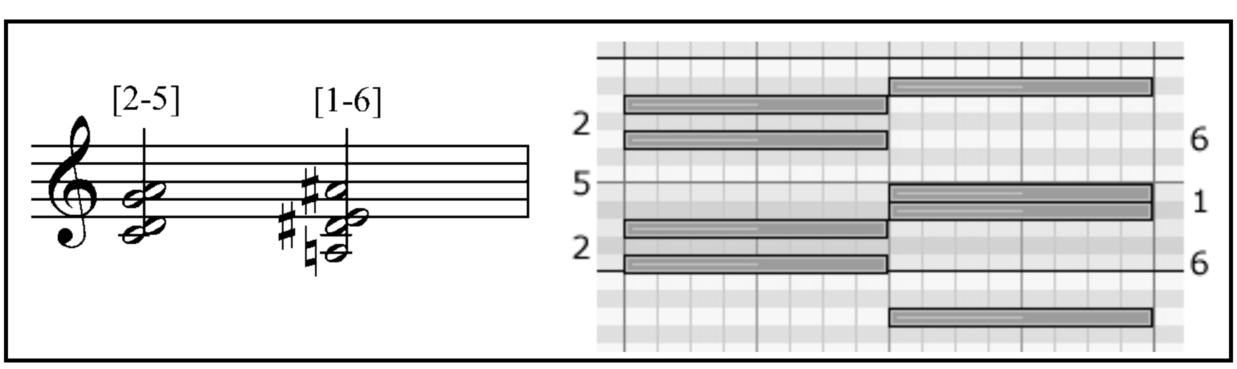


Figure 3.3. [2-5] moving in contrary motion to [1-6].

The three examples illustrated thus far have explored progressions involving a one-to-one partition mapping as well as a one-to-one cardinality mapping. In each case, both cycles have been partitions of 7 with cardinalities of 2. From this it follows that if two cycles have equivalent partitions and equivalent cardinalities, then they are compatible: they will consist of the same number of voices. The keen observer might ask, is this the only circumstance in which compatible voice leading may occur? What if two cycles have different partition intervals and cardinalities?

Figure 3.4 shows [1-4] leading to [1-3-1-5]. The first cycle is a partition of 5 of cardinality 2, and the second is a partition of 10 of cardinality 4. It should be obvious that the second set of numbers (10 and 4) is a multiple of the first set (5 and 2). To put it another way, the 1:2 partition ratio is equivalent to the 1:2 cardinality ratio. In order for mapping to occur, two

[15] Notational examples were created in *Finale 2007* and graphical examples were created in the music sequencing software *Logic Pro*.

stackings of [1-4] will be required for every single stacking of [1-3-1-5]. The LCM of 5 and 10 is 10, which is the interval that separates the cyclically adjacent pairs (C, A#), (Db, B), (F, D#), and (Gb, E). The voice movement here is a combination of similar, contrary, and sustained motion. The voice-leading distance traveled per LCM is 0 + 2 - 1 + 3 = 4, and the absolute distance per LCM is |0| + |2| + |-1| + |3| = 6, which is the maximum allowable: *p* – *c* = *d*, or 10 – 4 = 6.

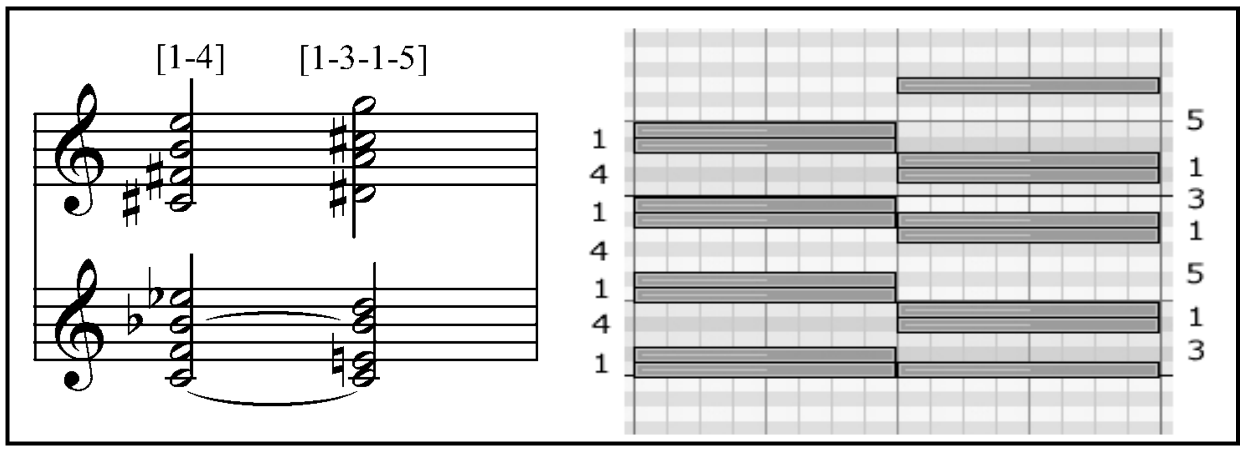


Figure 3.4. Progression from [1-4] to [1-3-1-5].

The relation by which voices are conserved may now be generalized: any cycle may map onto cycle X if its partition and cardinality are a multiple of cycle X, in other words, the partition ratio between two cycles must be equal to the cardinality ratio between them. Figure 3.5 (below) shows a graphical representation of six different progressions between cycles related by different ratios. From left to right these ratios are 1:2, 1:3, 1:4, 1:5, 2:3, and 2:5. In each case, the partition ratio is equivalent to the cardinality ratio, which is why voice conservation occurs.

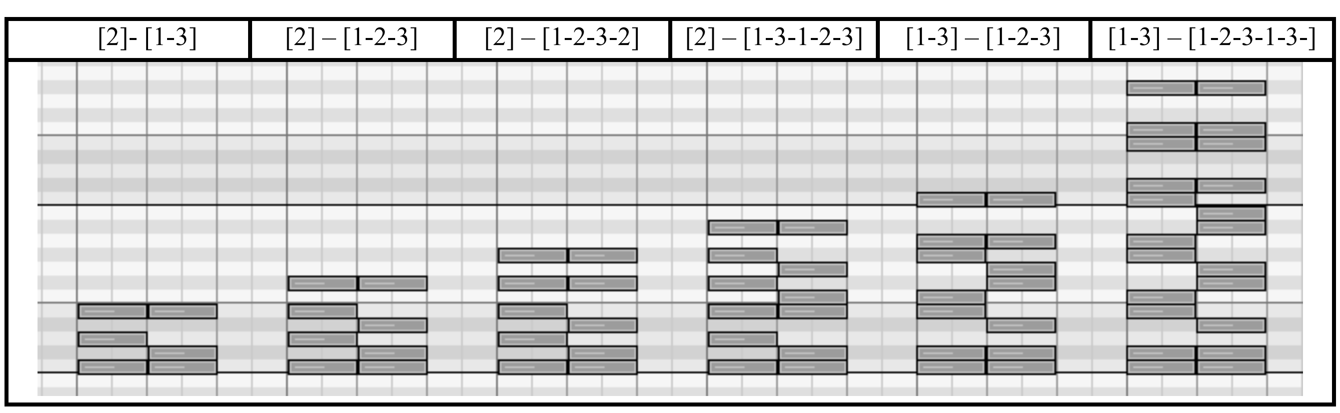


Figure 3.5. Voice conservation occurs when partition ratio and cardinality ratio are equivalent.

From this concept we can derive an array of cyclic compatibility shown in Table 3.1. Each row constitutes a *cyclic family* in which a cycle can map onto any other cycle in the same family without a loss of voice. In this Table, cycles have been generalized as cyclic classes. A *cyclic class* (cc) is a class of interval cycles that share a common partition *p* and a common cardinality *c* indicated by the designation *p-c*[16]. For instance, there are four possible realizations of cyclic class 6-3 (read a partition of six into three intervals), which are [1-1-4], [1-2-3], [1-3-2], and [2-2-2]. Cyclic families and cyclic classes are new concepts heretofore unknown in the music theory lexicon.

The first column shows cyclic classes whose partition and cardinality are *coprime*. These coprime classes are *irreducible*, thus serve as family archetypes from which cyclic families derive their name. All other classes within a family are multiples of this coprime class. Thus, 4-2 is a member of family 2-1, and 21-15 is a member of 7-5. The second column shows

[16] An alternative method for naming cyclic classes might be with the cardinality first and the partition interval (or list number) second, similar in vein to Allen Forte's listing of pitch classes. However, I believe that the method adopted here more appropriately reflects the importance of partition numbers for cycles in general. Remember that in this document cardinality refers to the number of intervals, not the number of pitches.

an initial cycle of each cyclic family class. The subsequent columns show classes that are members of the cyclic family related by a ratio (partition and cardinality) to the initial cycle.

It is possible to use this Table to set up relations by ratios other than the ones listed. An example of ratio 5:3 in the cyclic family 2-1 might involve [1-1-1-1-6] going to [1-1-4], or simply any member of cyclic class 10-5 going to 6-3. The chromatic scale is not included in this Table. If a cycle's partition number is equal to its cardinality, the result will simply be a chromatic scale [1]. For instance, cyclic class 4-4 is [1-1-1-1]. In addition, it is impossible for a cycle to have a cardinality higher than its partition number (such as 4-5), since this would divide the partition into intervals smaller than a semitone (or otherwise, the smallest unit used in a chosen tuning system). Note that some of the larger cyclic classes are unlikely to be used in a musical setting (for instance, one would rarely see a partition of 54 into 45 elements).

Table 3.1. Some Cyclic Families.

Cyclic Family:	Initial Cycle:	Cyclic classes listed by their relation to the initial cycle:									
		1:1	**2:1**	**3:1**	**4:1**	**5:1**	**6:1**	**7:1**	**8:1**	**9:1**	**10:1**
2-1	[2]	2-1	4-2	6-3	8-4	10-5	12-6	14-7	16-8	18-9	20-10
3-1	[3]	3-1	6-2	9-3	12-4	15-5	18-6	21-7	24-8	27-9	30-10
3-2	[1-2]	3-2	6-4	9-6	12-8	15-10	18-12	21-14	24-16	27-18	30-20
4-1	[4]	4-1	8-2	12-3	16-4	20-5	24-6	28-7	32-8	36-9	40-10
4-3	[1-1-2]	4-3	8-6	12-9	16-12	20-15	24-18	28-21	32-24	36-27	40-30
5-1	[5]	5-1	10-2	15-3	20-4	25-5	30-6	35-7	40-8	45-9	50-10
5-2	[1-4]	5-2	10-4	15-6	20-8	25-10	30-12	35-14	40-16	45-18	50-20
5-3	[1-1-3]	5-3	10-6	15-9	20-12	25-15	30-18	35-21	40-24	45-27	50-30
5-4	[1-1-1-2]	5-4	10-8	15-12	20-16	25-20	30-24	35-28	40-32	45-36	50-40
6-1	[6]	6-1	12-2	18-3	24-4	30-5	36-6	42-7	48-8	54-9	60-10
6-5	[1-1-1-1-2]	6-5	12-10	18-15	24-20	30-25	36-30	42-35	48-40	54-45	60-50
7-1	[7]	7-1	14-2	21-3	28-4	35-5	42-6	49-7	56-8	63-9	70-10
7-2	[1-6]	7-2	14-4	21-6	28-8	35-10	42-12	49-14	56-16	63-18	70-20
7-3	[1-1-5]	7-3	14-6	21-9	28-12	35-15	42-18	49-21	56-24	63-27	70-30
7-4	[1-1-1-4]	7-4	14-8	21-12	28-16	35-20	42-24	49-28	56-32	63-36	70-40
7-5	[1-1-1-1-3]	7-5	14-10	21-15	28-20	35-25	42-30	49-35	56-40	63-45	70-50
7-6	[1-1-1-1-1-2]	7-6	14-12	21-18	28-24	35-30	42-36	49-42	56-48	63-54	70-60

Before discussing the wider implications of cyclic families (see Chapter IV), more voice-leading issues should be examined. It should be self-evident that in most cases there are several potential paths from one vertically aligned cycle to another. A simple example of this is portrayed in Figure 3.6, in which every possible voice leading from one vertically aligned [2-5] cycle to another vertically aligned [2-5] cycle (cc 7-2) is shown. The voice leadings in this example consist of pairs (a, b), (c,d), and (e,f) that are retrograde-related, such that the first is descending in contour, and the second is ascending in contour. The voice-leading distances for these pairs (per LCM) are (-1, +1), (-3, +3), and (-5, +5). All-parallel motion is always excluded.

The composer often limits voice leading to a certain type depending on the context and character of a particular passage. For instance, the composer may wish to use only those voice leadings that result in: contrary motion (a and b), common tones (c and d), similar motion (e and f), ascent (a, c, and e), and/or descent (b, d, and f). Similarly, the composer may choose to confine his choices to voice leadings with the maximum or minimum: number of common tones, voice leading distance, number of voices moving by step, and so forth. Given that the number of voice leading options available between cycles with large partitions are often very large (especially cycles which also have small cardinalities), the need for imposing categorical restrictions is obvious. Generally speaking, voice leading incorporating stepwise and contrary motion will be more melodic in character.

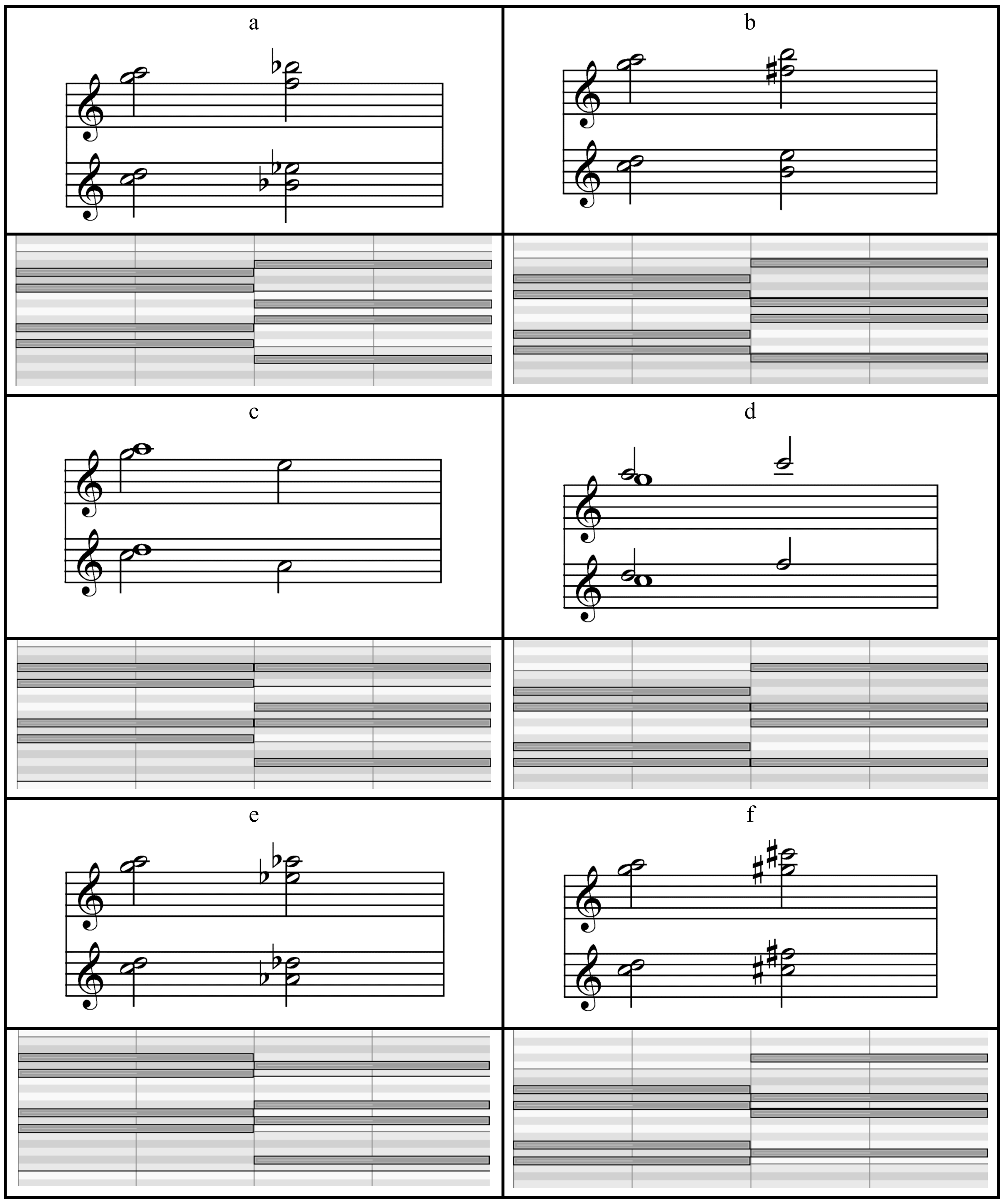


Figure 3.6. Potential voice leading paths from one [2-5] cycle to another [2-5] cycle.

Once the composer has selected the cycles he wishes to use, he may then arrange them into chord progressions. This can be done quite systematically. An ordered progression of vertically aligned cycles is called a *metacycle*. Metacycles infuse cohesion and interest into a musical passage. Analogous to interval cycles, *metacyclic completion* occurs when the initial and terminal cycles are identical (the same cycle) and in the same rotation (chordal inversion). The terminal cycle may or may not be transposed, depending on voice leading. The term 'rotation' will henceforth be used in place of 'chordal inversion' when referring to metacycles, whereby *rotation 0* replaces 'root position,' *rotation 1* replaces 'first inversion,' and so forth. The number of rotations possible for any given metacycle is equal to the cardinality of the cycles per LCM. A number of elements are retained in each repetition of a metacycle: the interval cycles used, the order of those interval cycles, and the voice leading path(s). This creates multiple levels of invariance and thus coherence. The number of voice leading paths is always one less than the number of distinct cycles within a metacycle. For instance, in Figure 3.7, there are three metacycles, thus two voice-leading paths. Metacycles must consist of a minimum of two cycles embedding a single voice leading path duplicated every repetition of the metacycle. These two cycles may be (and often are) transpositionally equivalent. In other words a [4-5] cycle may move to another (transposed and rotated) [4-5] cycle. Conversely, there is no upper limit placed on the number of distinct cycles and voice-leading paths, but as a general rule, listeners are more apt to discern metacycles of shorter length.

Optimal metacycles repeat the maximum number of times before metacyclic completion occurs. This takes place when the first repetition of the metacycle is rotated. Voice-leading distance and motion often determine whether or not a metacycle is optimal. If in the same rotation, the metacycle will simply be transposed upon repetition, thus will not be optimized. The maximum number of times a metacycle is able to repeat before transposition is equivalent to the cardinality per LCM. For example, a metacycle consisting of the interval cycles [4-5] and [2-7] is able to repeat a maximum of two times since the cardinality per LCM is 2. Figure 3.7 shows a non-optimized metacycle consisting of [4-5] and [2-7]. Since the first repetition of the initial cycle is in the exact same rotation as the first cycle, any further repetitions will be simply transpositions: both cycles are in the [4-5] position (which is called rotation 0).

The first instance of any given metacycle is labeled T0 (transposition 0). Subsequent transpositions are labeled in accordance with their transpositional relation to the initial instance by a positive or negative number of semitones. The + and - signs are used due to the preservation of interval locations in pitch space, and semitone size is literally determined rather than mod 12.

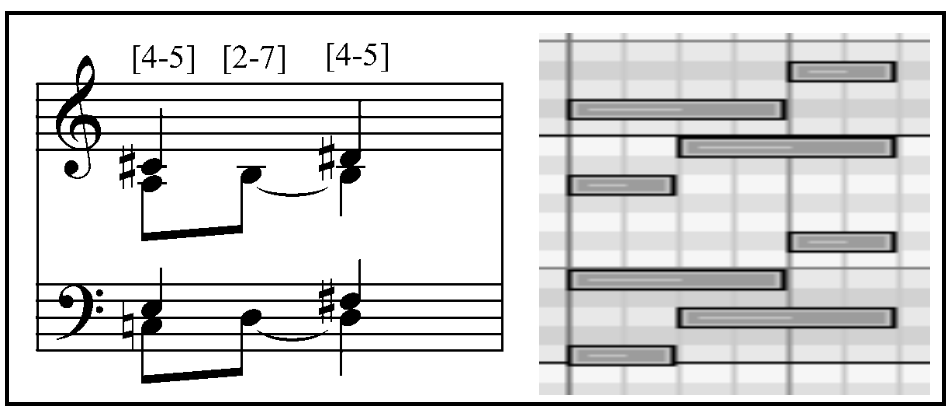


Figure 3.7. Non-optimized metacycle consisting of [4-5] and [2-7].

Figure 3.8 shows an optimal metacycle. In this case, the first repetition of the initial cycle is rotated to [5-4] (called rotation 1), so the second repetition is not simply a transposition. A different voice leading distance between the second and third cycles accounts for the optimal outcome. In this case, the distance is 3, whereas in the non-optimized version, the distance is 2. The number of rotations possible for any given metacycle is equal to the cardinality of the cycles per LCM. In this instance two rotations are possible: rotation 0 and rotation 1.

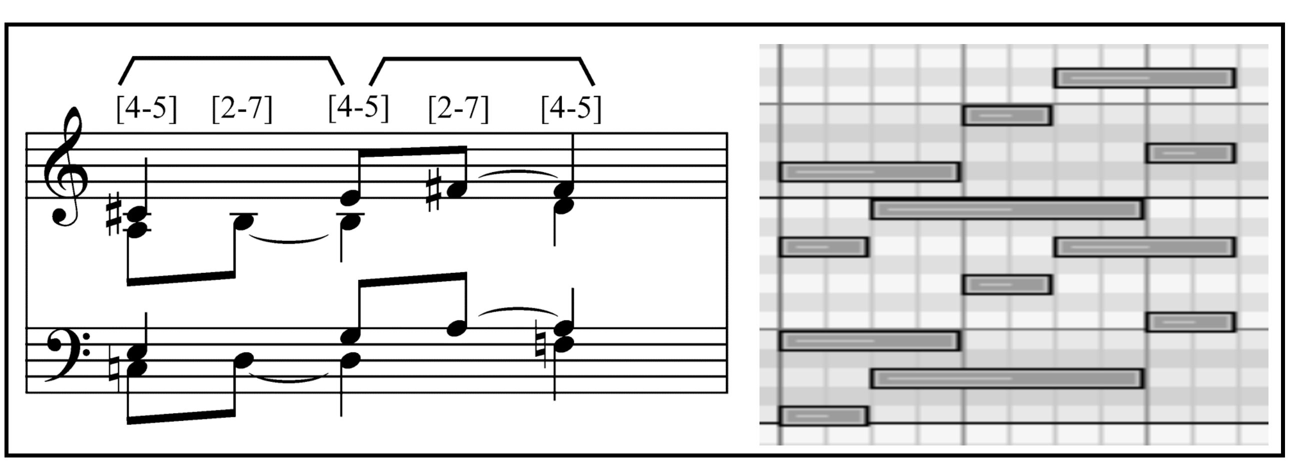


Figure 3.8. Optimal metacycle consisting of [4-5] and [2-7].

A very interesting type of metacycle is one that results in canonic imitation. For this to occur, the absolute (+ or –) voice-leading distance within one metacycle must sum to the LCM of the partition intervals of the cycles. Figure 3.9 (below) shows a metacycle consisting of cycles [5-11], [2-14], and [3-13] (cc 16-2). The numbers in the example represent canonic entrances. Since the voice-leading distance per LCM, per metacycle is identical to the partition interval, (3 + 0 + 2 + 0 + 11 + 0 = 16), canonic imitation occurs. To put it another way, the horizontal motion of each voice outlines the same [3-2-11] cycle (cc

16-3). Therefore, it is clearly not necessary for the cardinalities of the vertical and horizontal cycles to be equal for canonic imitation to occur. The horizontal and vertical constructions need only partition the same interval or their partition intervals must be non-coprime. Canonic imitation may occur regardless of whether or not the metacycle is optimal. In this case, it is optimal. So long as the total non-absolute voice-leading is equal or non-coprime to the partitions of the cycles, it does not necessarily have to be in similar motion. It may include contrary and sustained motion as well.

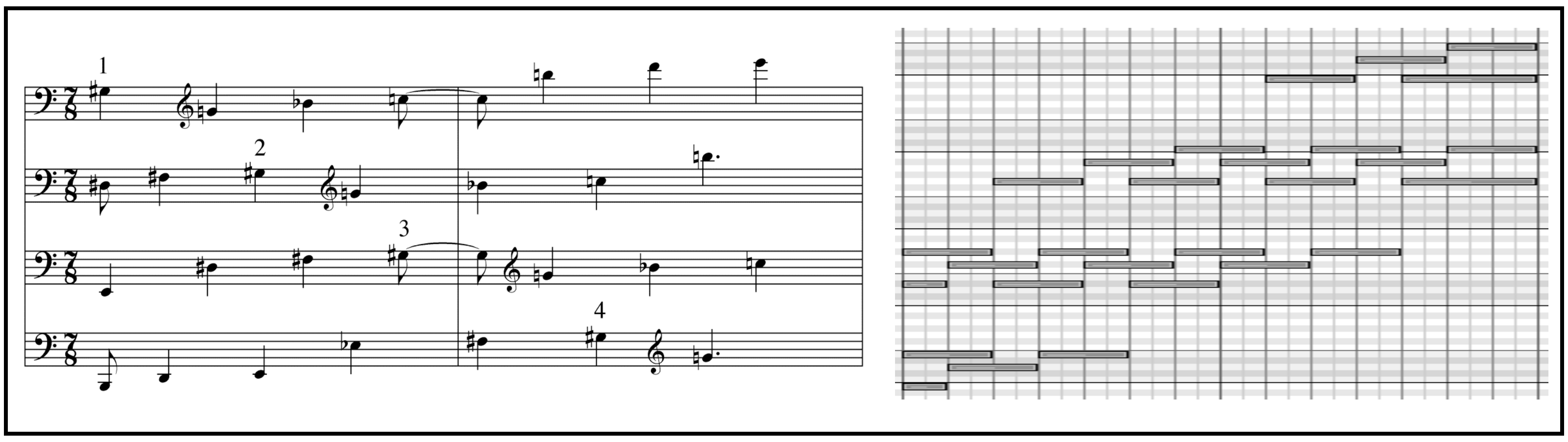


Figure 3.9. Canonic metacycle of vertical cc 16-2 cycles and horizontal cc 16-3 cycles.

In the composition *Symphony No. 1*, metacycles are often constructed first; independent from the final result. Subsequently, they are utilized as ‘referential constructions’ such that they are treated in a flexible manner when orchestrated. As chapter V will clarify, the ‘voices’ from a referential construction often do not resemble the instrumental ‘lines.’ Examples of compound melodies, arpeggiations, and voice crossings can readily be found, while the integrity of the original construction – its pitch-class content, regristral placements, and so on – remain intact.

CHAPTER IV
CYCLIC FAMILY DISTANCE AND EQUIVALENCE RELATIONS

To use the tonal analogy, cyclic families may be regarded as keys. Just as the distance between two keys is measured by the number of tones different between them, the distance between two cycles may be measured by the number of unique voices between them per LCM. We may call this measure of distance *cyclic family distance*. The most closely-related keys have only a single note difference, for example C major and F major (B versus Bb). Similarly, the most closely-related cycles differ by only a single voice. In fact, successions of closely-related cycles occur quite frequently in tonal music. For example, triads (cc 12-3) often lead to seventh chords (cc 12-4) and vice versa despite belonging to different cyclic families (refer back to Figure 2.2). While most of the progressions in *Symphony No. 1* are limited to cyclic families, thinking of them as keys opens up the possibility of 'modulating' between them.

When two cycles partition the same interval (a partition ratio of 1:1), their distance is determined by the difference of their cardinalities. For example, [5] and [1-4] differ by one voice, [5] and [1-1-3] differ by two voices, and so forth. This is the type of distance that exists between chords of various cardinalities within tonal music inasmuch as all tonal structures are partitions of 12. Unfortunately, the converse isn't always true: two cycles sharing a common cardinality (a cardinality ratio of 1:1), will not necessarily differ by the extent to which their partitions differ. For cycles of cardinality 1, it is true. For example [5] and [6] differ by a single voice. In fact, a pair of cycles with cardinality of 1 will differ by one voice if their partitions are related by the ratios listed in Table 4.1. Each row in Table 4.1 represents a group of closely-related monad cycles (cardinality 1).

Table 4.1. Some Closely-Related Cycles of Cardinality 1 with a Distance of 1.

Cycles of Cardinality 1:	Closely-related cycles of cardinality 1:																	
	1:2	**2:1**	**2:3**	**3:2**	**3:4**	**4:3**	**4:5**	**5:4**	**5:6**	**6:5**	**6:7**	**7:6**	**7:8**	**8:7**	**8:9**	**9:8**	**9:10**	**10:9**
2	1	4	—	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—
3	—	6	2	—	—	4	—	—	—	—	—	—	—	—	—	—	—	—
4	2	8	—	6	3	—	—	5	—	—	—	—	—	—	—	—	—	—
5	—	10	—	—	—	—	4	—	—	6	—	—	—	—	—	—	—	—
6	3	12	4	9	—	8	—	—	5	—	—	7	—	—	—	—	—	—
7	—	14	—	—	—	—	—	—	—	—	6	—	—	8	—	—	—	—
8	4	16	—	12	6	—	—	10	—	—	—	—	7	—	—	9	—	—
9	—	18	6	—	—	12	—	—	—	—	—	—	—	—	8	—	—	10
10	5	20	—	15	—	—	8	—	—	12	—	—	—	—	—	—	9	—
11	—	22	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
12	6	24	8	18	9	16	—	15	10	—	—	14	—	—	—	—	—	—
13	—	26	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
14	7	28	—	21	—	—	—	—	—	—	12	—	—	16	—	—	—	—
15	—	30	10	—	—	20	12	—	—	18	—	—	—	—	—	—	—	—
16	8	32	—	24	12	—	—	20	—	—	—	—	14	—	—	18	—	—
17	—	34	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
18	9	36	12	27	—	24	—	—	15	—	—	21	—	—	16	—	—	20
19	—	38	—	—	—	—	—	—	—	—	—	—	—	—	—	—	22	—
20	10	40	—	30	15	—	16	25	—	24	—	—	—	—	—	—	—	—

Reading Table 4.1, we can see that cycle [6] may modulate to the closely-related cycles [3], [12], [4], [9], [8], [5], or [7] with a disparity of only one voice. This is confirmed in Figure 4.1 below.

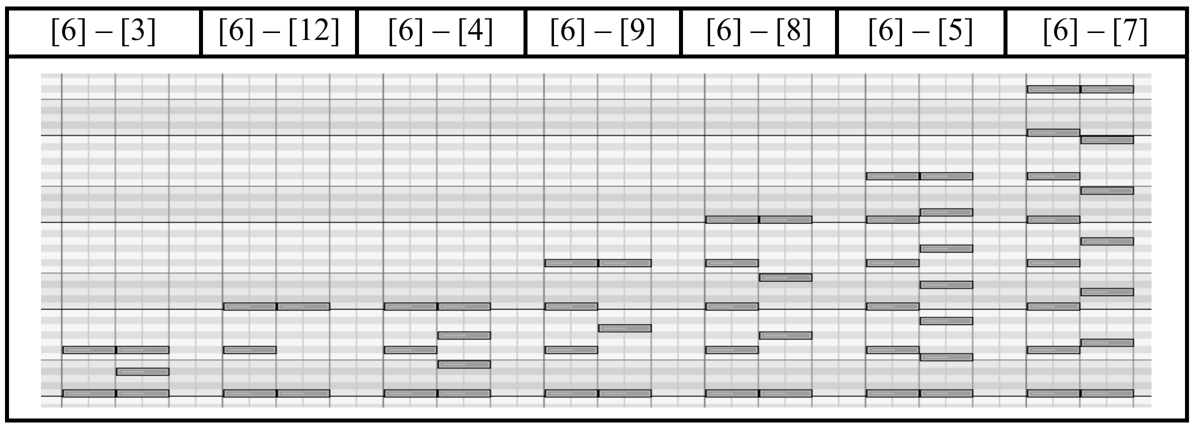


Figure 4.1. Modulation from monad cycles to other, closely-related monad cycles.

If the first cycle is a monad (cardinality of 1), and the second cycle is a dyad (cardinality of 2), they will differ by only one voice if their partitions are related by the ratios listed in Table 4.2. Again, each row is a group of closely-related monad cycles.

Table 4.2. Some Closely-Related Cycles of Cardinality 1 going to Cardinality 2.

Cycles of Cardinality 1	Closely-related partitions of cardinality 2																	
	1:1	3:1	3:2	5:2	5:3	7:3	7:4	9:4	9:5	11:5	11:6	13:6	13:7	15:7	15:8	17:8	17:9	19:9
[2]	—	6	3	5	—	—	—	—	—	—	—	—	—	—	—	—	—	—
[3]	3	9	—	—	5	7	—	—	—	—	—	—	—	—	—	—	—	—
[4]	4	12	6	10	—	—	7	9	—	—	—	—	—	—	—	—	—	—
[5]	5	15	—	—	—	—	—	—	9	11	—	—	—	—	—	—	—	—
[6]	6	18	9	15	10	14	—	—	—	—	11	13	—	—	—	—	—	—
[7]	7	21	—	—	—	—	—	—	—	—	—	—	13	15	—	—	—	—
[8]	8	24	12	20	—	—	14	18	—	—	—	—	—	—	15	17	—	—
[9]	9	27	—	—	15	21	—	—	—	—	—	—	—	—	—	—	17	19
[10]	10	30	15	25	—	—	—	—	18	22	—	—	—	—	—	—	—	—
[11]	11	33	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
[12]	12	36	18	30	20	28	21	27	—	—	22	26	—	—	—	—	—	—
[13]	13	39	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
[14]	14	42	21	35	—	—	—	—	—	—	—	—	26	30	—	—	—	—
[15]	15	45	—	—	25	35	—	—	27	33	—	—	—	—	—	—	—	—
[16]	16	48	24	40	—	—	28	36	—	—	—	—	—	—	30	34	—	—
[17]	17	51	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
[18]	18	54	27	45	30	42	—	—	—	—	33	39	—	—	—	—	34	38
[19]	19	57	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
[20]	20	60	30	50	—	—	35	45	36	44	—	—	—	—	—	—	—	—

Reading Table 4.2, we can see that cycle [3] may modulate to the closely-related cycles [1-2], [1-8], [1-4], or [1-6] with a disparity of only one voice. This is confirmed in Figure 4.2.

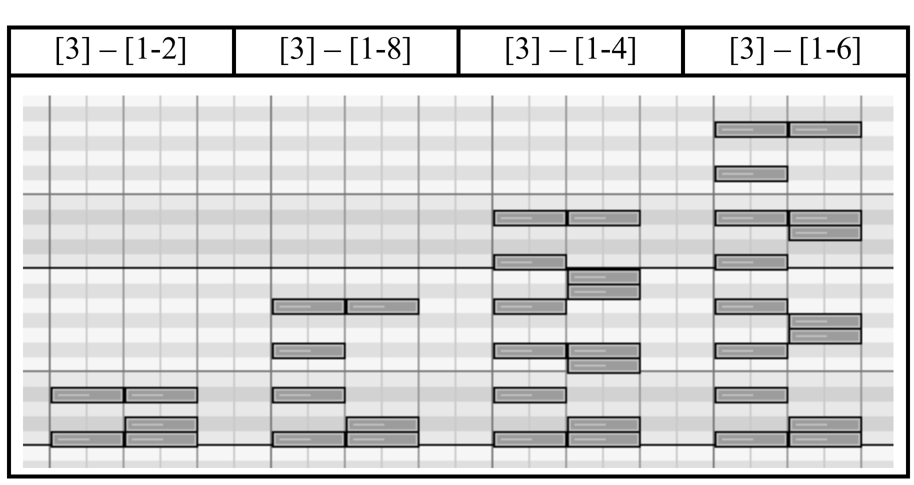


Figure 4.2 Modulation from monad cycles to closely-related dyad cycles.

At this point, it might be helpful to backtrack and examine how these ratios are determined. For example, if we are given one cycle or cyclic class, how do we quickly find another cycle that is at a given distance? (Remember that distance is measured by the difference in voices between a pair of cycles per least common multiple.) For instance, what cycle is at a distance of ± 1 from cycle [1-5-5] (cc 11-3)? In order to solve this problem, it is necessary to use the linear Diophantine equation $\boldsymbol{ax - by = \pm d}$ where cyclic class *a-y* maps on to cyclic class *b-x* which are related by a partition ratio of *a:b* and a cardinality ratio of *y:x* at a distance of *d*. In this equation, every value other than distance must be a positive integer. First plug in the numbers from cycle [1-5-5], which gives us $11x - b3 = \pm 1$. Now plug in a value for *x* or *b* to solve the equation. For instance if we let $b = 4$, then $x = 1$:

$$\begin{array}{cccc} ax & - & by & = \pm d \\ (11 * 1) & - & (4 * 3) & = \pm 1 \\ 11 & - & 12 & = \pm 1 \end{array}$$

From this it follows that the cyclic class 11-3 (*a-y*) is at a distance of ± 1 from cyclic class 4-1 (*b-x*). However, this is only one of an infinite number of solutions for this equation. In fact, if a linear equation can be solved, there are always an infinite number of solutions. As stated, we are only interested in those that are integers. Another solution is $b = 7$ and $x = 2$:

$$\begin{array}{cccc} ax & - & by & = \pm d \\ (11 * 2) & - & (7 * 3) & = \pm 1 \\ 22 & - & 21 & = \pm 1 \end{array}$$

Thus, cyclic class 11-3 (*a-y*) is at a distance of ± 1 from cyclic class 7-2 (*b-x*).

Since there are an infinite number of solutions, we have to choose those that are most practical for a musical application. In the first solution, a single mapping occurs at the interval of 44, which is the least common multiple of the partitions 11 and 4. This is a very large interval for a single mapping, especially considering it is ideal to have two mappings presented in the music, and yet it is not nearly as large as the second solution in which a single mapping occurs at the interval 77. The next solution after that ($b = 15$ and $x = 4$) has a LCM of 165 with cyclic class 11-3. Clearly, the solution chosen will have to take into consideration the range of the ensemble used, as well as the type of tuning employed. In this case, the first solution would seem to be the obvious choice. Figure 4.3 shows the first two solutions realized as cyclic progressions.

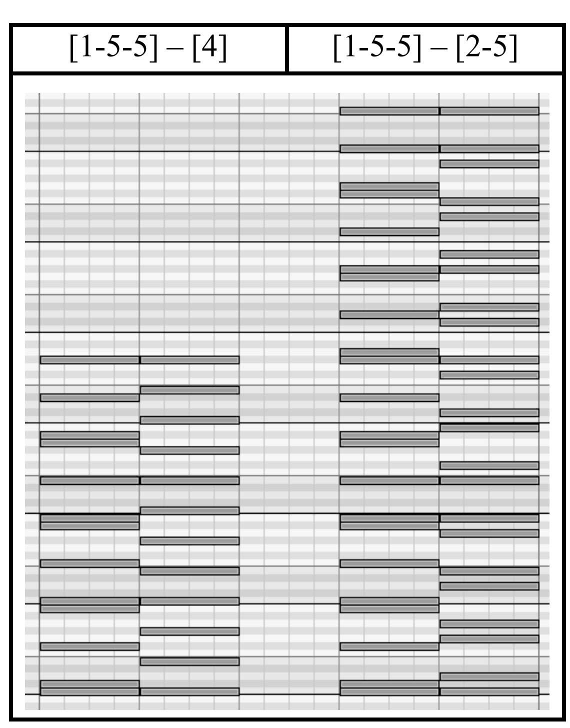


Figure 4.3. The first two solutions for $11x - b3 = \pm 1$ realized as cyclic progressions.

We can of course solve for any variable, depending on the situation. Consider another example: what is the distance between cycles [1-2-4] and [1-9]? First, it is helpful to convert these cycles into the cyclic class designations 7-3 and 10-2.

Knowing that *a-y* is the cyclic class of the first cycle and *b-x* is the cyclic class of the second cycle, we simply plug those values into the equation:

$$\begin{array}{cccc} ax & - & by & = \pm d \\ (7 * 2) & - & (10 * 3) & = \pm d \\ 14 & - & 30 & = -16 \end{array}$$

Thus, the distance between [1-2-4] and [1-9] is 16. We can find the distance between any given pair of cycles in this manner.

Consider another question: what is the partition ratio *a:b* when the cardinality ratio *y:x* is 4:9 and the distance is ± 1? Simply plug in the given values:

$$\begin{array}{cccc} ax & - & by & = \pm d \\ a9 & - & b4 & = \pm 1 \\ (1 * 9) & - & (2 * 4) & = \pm 1 \\ 9 & - & 8 & = 1 \end{array}$$

If $a = 1$, then $b = 2$, thus a pair of cycles with cardinality 4:9 will be at a distance of ± 1 if their partition ratio is 1:2. Again, this is only one of an infinite number of solutions. This gives us the cyclic classes 1-4 and 2-9. Note that these cyclic classes are actually impossible, since you cannot divide a partition of 1 into 4 elements, nor can you divide a partition of 2 into 9 elements. This problem, however, is easily resolved, since you can freely multiply the partition ratio without affecting distance provided the cardinality ratio is unaltered [17]. For example, if we multiply the partition ratio by 10, we get the partition ratio 10:20 and the cyclic classes 10-4 and 20-9. In this case, [1-3-1-5] and [1-3-1-2-3-1-3-1-5] should differ by one voice. This is clearly evident in Figure 4.4. Since the LCM is 20, [1-3-1-5] must have two stackings, (eight intervals) for every one stacking (nine intervals) of [1-3-1-2-3-1-3-1-5], which when subtracted is (4 * 2) – (9 * 1) = - 1.

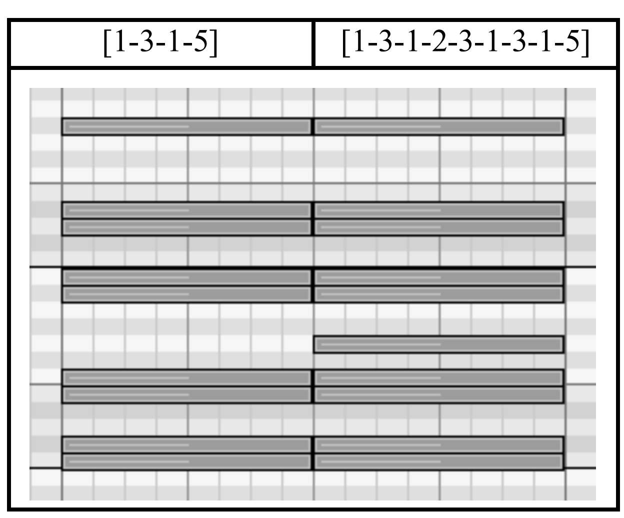


Figure 4.4. Two stackings of [1-3-1-5] going to one stacking of [1-3-1-2-3-1-3-1-5].

Note that if we were to reverse course and calculate the distance between cyclic class 10-4 and 20-9, that distance wouldn't be 1:

$$\begin{array}{lcccc} & ax & - & by & = \pm d \\ \text{Incorrect:} & (10 * 9) & - & (20 * 4) & = \pm d \\ & 90 & - & 80 & = 10 \end{array}$$

This is clearly incorrect, since we've already determined that the distance is 1. Thus, before the distance between a given pair of cycles can be determined, the LCM of *a* and *b* must first be divided by *a* and *b*. In this case, the LCM (10, 20) = 20, so:

$$a = 20 \div 10 = 2$$

$$b = 20 \div 20 = 1$$

[17] Conversely, multiplying the cardinality ratio will result in a change of distance if the partition ratio is unaltered.

Table 4.3 shows some solutions of $ax - by = \pm d$ expressed as ratios for cycles of cardinality 1 going to cycles of up to cardinality 11, and at a distance of up to ± 3. The columns in Table 4.3 correspond to a single cardinality ratio listed in the uppermost row. The subsequent rows in Table 4.3 show partition ratios at different distances for each respective cardinality ratio. For example, to go from a cycle of cardinality 7 to a cycle of cardinality 1 with a difference of one voice, one could use a partition ratio of 6:1. Again, since a partition of 6 cannot be divided into 7 parts, it is necessary to multiply the partition ratio by some number. If we multiply it by 2, then we get a partition ratio of 12:2 (the cardinality ratio 7:1 does not change), so cycle [1-2-2-1-2-1-3] going to [2] would be one workable example. For distances between cycles of other cardinalities, see the Appendix B.

Table 4.3. Some Distances between Cycles of Cardinality 1 going to Cycles of up to Cardinality 11.

Distance:	**Cardinality Ratios:**															
	1:1				**2:1**		**3:1**		**4:1**	**5:1**	**6:1**	**7:1**	**8:1**	**9:1**	**10:1**	**11:1**
	Partition Ratios:															
±0	1:1				2:1		3:1		4:1	5:1	6:1	7:1	8:1	9:1	10:1	11:1
±1	2:1	7:6	12:11	17:16	1:1	11:6	2:1	17:6	3:1	4:1	5:1	6:1	7:1	8:1	9:1	10:1
	1:2	6:7	11:12	16:17	3:1	13:6	4:1	19:6	5:1	6:1	7:1	8:1	9:1	10:1	11:1	12:1
	3:2	8:7	13:12	18:17	3:2	13:7	5:2	20:7	7:2	9:2	11:2	13:2	15:2	17:2	19:2	21:2
	2:3	7:8	12:13	17:18	5:2	15:7	7:2	22:7	9:2	11:2	13:2	15:2	17:2	19:2	21:2	23:2
	4:3	9:8	14:13	19:18	5:3	15:8	8:3	23:8	11:3	14:3	17:3	20:3	23:3	26:3	29:3	32:3
	3:4	8:9	13:14	18:19	7:3	17:8	10:3	25:8	13:3	16:3	19:3	22:3	25:3	28:3	31:3	34:3
	5:4	10:9	15:14	20:19	7:4	17:9	11:4	26:9	15:4	19:4	23:4	27:4	31:4	35:4	39:4	43:4
	4:5	9:10	14:15	19:20	9:4	19:9	13:4	28:9	17:4	21:4	25:4	29:4	33:4	37:4	41:4	45:4
	6:5	11:10	16:15	21:20	9:5	19:10	14:5	29:10	19:5	24:5	29:5	34:5	39:5	44:5	49:5	54:5
	5:6	10:11	15:16	20:21	11:5	21:10	16:5	31:10	21:5	26:5	31:5	36:5	41:5	46:5	51:5	56:5
±2	3:1	5:7	10:12	15:17	4:1	14:6	1:1	16:6	2:1	3:1	4:1	5:1	6:1	7:1	8:1	9:1
	4:2	9:7	14:12	19:17	2:2	12:7	5:1	20:6	6:1	7:1	8:1	9:1	10:1	11:1	12:1	13:1
	1:3	6:8	11:13	16:18	6:2	16:7	4:2	19:7	6:2	8:2	10:2	12:2	14:2	16:2	18:2	20:2
	5:3	10:8	15:13	20:18	4:3	14:8	8:2	23:7	10:2	12:2	14:2	16:2	18:2	20:2	22:2	24:2
	2:4	7:9	12:14	17:19	8:3	18:8	7:3	22:8	10:3	13:3	16:3	19:3	22:3	25:3	28:3	31:3
	6:4	11:9	16:14	21:19	6:4	16:9	11:3	26:8	14:3	17:3	20:3	23:3	26:3	29:3	32:3	35:3
	3:5	8:10	13:15	18:20	10:4	20:9	10:4	25:9	14:4	18:4	22:4	26:4	30:4	34:4	38:4	42:4
	7:5	12:10	17:15	22:20	8:5	18:10	14:4	29:9	18:4	22:4	26:4	30:4	34:4	38:4	42:4	46:4
	4:6	9:11	14:16	19:21	12:5	22:10	13:5	28:10	18:5	23:5	28:5	33:5	38:5	43:5	48:5	53:5
	8:6	13:11	18:16	23:21	10:6	20:11	17:5	32:10	22:5	27:5	32:5	37:5	42:5	47:5	52:5	57:5
±3	4:1	10:7	15:12	20:17	5:1	15:6	6:1	21:6	1:1	2:1	3:1	4:1	5:1	6:1	7:1	8:1
	5:2	5:8	10:13	15:18	1:2	11:6	3:2	18:7	7:1	8:1	9:1	10:1	11:1	12:1	13:1	14:1
	6:3	11:8	16:13	21:18	7:2	17:7	9:2	24:7	5:2	7:2	9:2	11:2	13:2	15:2	17:2	19:2
	1:4	6:9	11:14	16:19	3:3	13:7	6:3	21:8	11:2	13:2	15:2	17:2	19:2	21:2	23:2	25:2
	7:4	12:9	17:14	22:19	9:3	19:8	12:3	27:8	9:3	12:3	15:3	18:3	21:3	24:3	27:3	30:3
	2:5	7:10	12:15	17:20	5:4	15:8	9:4	24:9	15:3	18:3	21:3	24:3	27:3	30:3	33:3	36:3
	8:5	13:10	18:15	23:20	11:4	21:9	15:4	30:9	13:4	17:4	21:4	25:4	29:4	33:4	37:4	41:4
	3:6	8:11	13:16	18:21	7:5	17:9	12:5	27:10	19:4	23:4	27:4	31:4	35:4	39:4	43:4	47:4
	9:6	14:11	19:16	24:21	13:5	23:10	18:5	33:10	17:5	22:5	27:5	32:5	37:5	42:5	47:5	52:5
	4:7	9:12	14:17	19:22	9:6	19:10	15:6	30:11	23:5	28:5	33:5	38:5	43:5	48:5	53:5	58:5

Distance charts such as Table 4.3 are valuable tools in planning out a composition or passage of cycles and cyclic relations. For instance, one might saturate a musical passage with some invariant feature, such as an invariant distance, partition, or cardinality. For example, if one wanted to maintain a partition ratio of 4:1, one could potentially modulate through all of the cardinality ratios listed above regardless of distance, since the partition ratio 4:1 can be found in several (if not all) cardinality ratios (columns) at different distances. An interesting textural experiment might involve starting with a widely-spaced cycle of few voices and gradually build up to a dense cycle with many voices by constantly moving to cycles a distance of +1 (or vice versa). Such modulations toward increasing texture might be placed at certain intervals between passages consisting of distance 0 progressions (cyclic family progressions). Moreover, this interval of time could itself be precisely controlled.

A simple example of cardinality and distance invariance is shown in Figure 4.5 in the form of a palindromic progression. All of the cycles in this example are cardinality 1 and are at a distance of ± 1 from cycles immediately adjacent to them. The progression is [2] – [3] – [4] – [5] – [6] – [7] – [8] – [9] – [10] – [11] – [12], followed by the same pattern in reverse. The partition ratios are 2:3, 3:4, 4:5, 5:6, …11:12, and in reverse 12:11, 11:10, …3:2 (see Table 4.3). An interesting fact about this alignment is that horizontal voice movement consists of a sustained voice in the base moving by 0, a chromatic voice moving by 1, a whole-tone voice moving by 2, and so forth, up to a soprano voice moving by quartal skips 5. I have artificially limited the voices to six, regardless of LCM between cycles, thus, voice difference per LCM is not perceivable. Thus, after cycle [5], it may not be readily apparent that cycles are able to map onto each other. This sort of manipulation of the system is wholly desirable. This statement reflects the fact that a system should begin as a starting foundation from which we may freely diverge, building upon or taking away from it depending upon the demands of musical and aesthetic aims. It should not ignore these aims in deference to strict adherence. The primary benefit of this type of divergence is that parallel motion inherent between cyclical adjacencies is eliminated, although still theoretically implied.

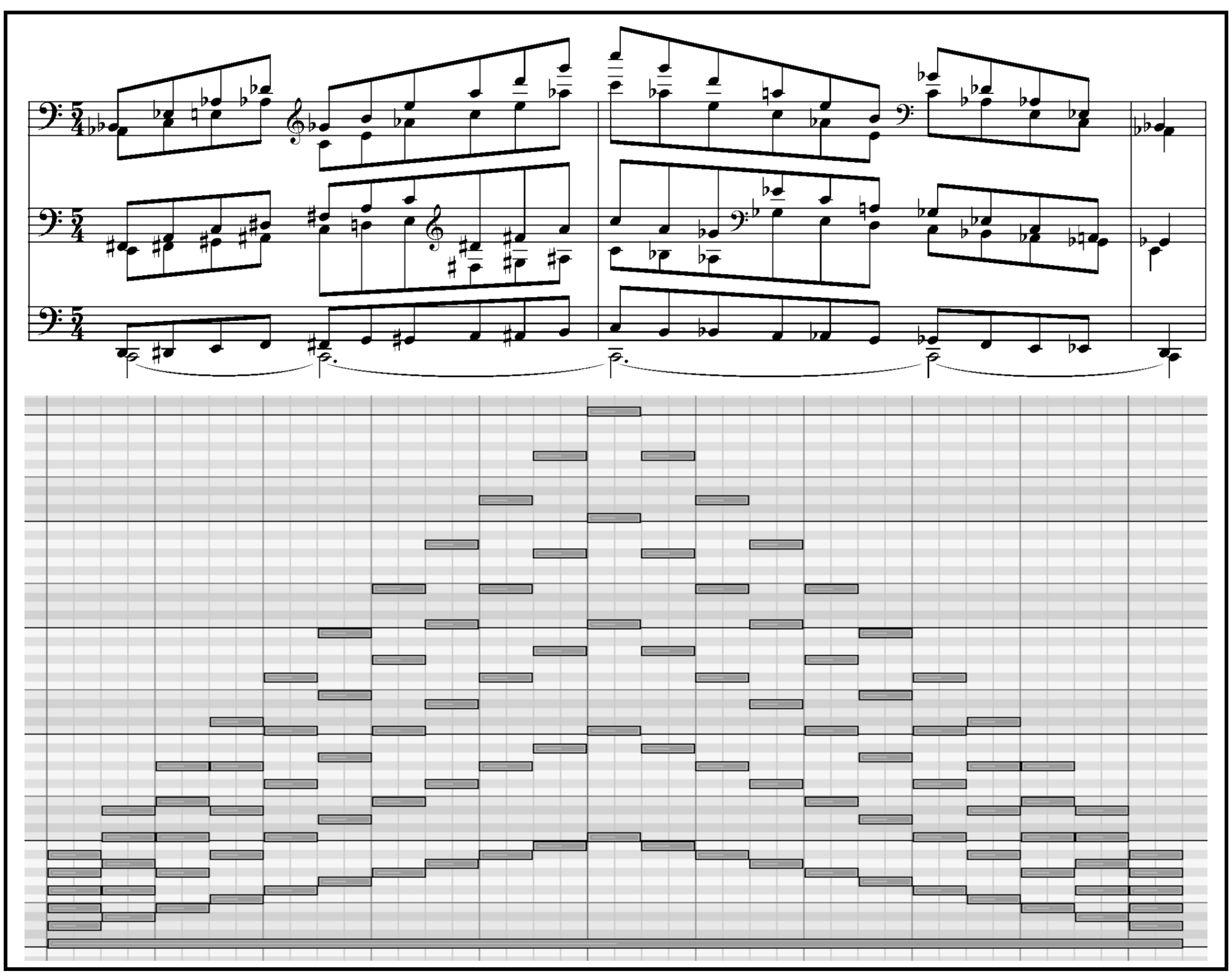


Figure 4.5. Cardinality and distance invariance in the form of a palindromic metacycle.

There are other ways to group cycles that may serve as alternatives or counterparts to cyclic families. First, we can group together cyclic families whose partitions conform to the same partition regardless of cardinality, such as 7-1, 7-2, 7-3, 7-4, and so forth. We may call this sort of invariance a *partition class*. This broadens the number of cycles available for a composition, movement, or passage without sacrificing coherence and unity within that formal unit, and makes intuitive sense. As it turns out, there is already a major precedent for partition-class invariance: in tonal music, every construction is a member of partition class 12. Thus, triads (cc 12-3) and seventh chords (cc 12-4) are grouped together.

Furthermore, it is possible to group cycles based upon an even broader relation: the multiple relation. This relation is not limited to a specific cardinality relation as is the case in cyclic families, thus it encompasses a larger set of cycles. The smallest numbers that can’t be divided within a multiple relation are prime numbers. Thus, we may call a specific multiple relation a *prime-number class (or PNC)*: a class of partitions the highest prime factors of which are equivalent to a specific prime number. If a nonprime has only one prime factor, it is easily assigned to a prime-number class. For example, 2, 4, 8, 16,

and 32 have only one prime factor (2), so they are placed into prime-number class 2. When a nonprime has more than one prime factor, it is placed into the class corresponding to the highest of those prime factors. Table 4.4 shows some examples of this procedure.

Table 4.4. Some Partitions, Prime Factors, and Prime-Number Classes.

Partition:	Prime factors:	Prime-number Group (highest prime factor):
6	2 * 3	3
7	(1) * 7	7
14	2 * 7	7
24	2 * 2 * 2 * 3	3
30	2 * 3 * 5	5
32	2 * 2 * 2 * 2 * 2	2

In Table 4.5, each column represents a single prime-number class. The first row is the name of each prime-number class. The middle row shows the most closely related nonprime numbers whose only prime factor corresponds to the prime-number class. The bottom row shows the more remotely related nonprime numbers whose highest prime factor corresponds to the prime-number class. An exclusive set of prime-number classes is assigned to each movement within *Symphony No. 1*, thus creating coherence on a broad formal level. In such a case, cyclic families and the distances between them function on a more localized level of form.

Table 4.5. Some Prime-Number Classes.

Prime-number classes:		2	3	5	7	11	13	17	19
Partitions:	Single prime factor: *n* < 100	2, 4, 8, 16, 32, 64	3, 9, 27, 81	5, 25	7, 49	11	13	17	19
	Highest prime factor: *n* < 100	none	6, 12, 18, 24, 36, 48, 54, 72, 96	10, 15, 20, 30, 40, 45, 50, 60, 75, 80, 90, 100	14, 21, 28, 35, 42, 56, 63, 70, 84, 98	22, 33, 44, 55, 66, 77, 88, 99	26, 39, 52, 65, 78, 91	34, 51, 68, 85	38, 57, 76, 95

Figure 4.6 illustrates the equivalence classes implicit in this compositional method, going from the broad to increasingly more narrow cases. The idea of equivalence relations can be expressed as follows: often it is convenient to group together objects having some common property and consider them all in an identical way (for example, the fractions 4/2 and 2/1). Formally, it would be incorrect to simply ‘declare them to be equal,’ since equality alludes to something much more precise. Instead, the appropriate notion is that of an equivalence relation, whereby we can call such things equivalent. Within the set of all musical constructions, we may consider as an equivalence class the set of all interval cycles: ordered patterns of intervals fixed in pitch space. Within that broad framework lies the still broad, but considerably more limited prime-number class, whereby 7-1 ~ 7-3 ~ 14-5 ~ 21-9. The symbol ~ means “is equivalent to.” Within the prime-number class lies the partition class, whereby the cyclic classes 7-1 ~ 7-2 ~ 7-3 ~ 7-4. Next is the cyclic-family class, whereby the cyclic classes 7-2 ~ 14-4 ~ 56-16 within cyclic family 7-2. Clearly, members of a cyclic family may intersect (or be a member of) two or more prime-number classes and/or partition classes. This is especially the case for cyclic families 2-1, 3-1, and 3-2. The next equivalence class is cyclic class, whereby interval cycles [1-6] ~ [2-5] ~ [3-4] within cyclic class 7-2. The most specific of the equivalence classes is simply a particular interval cycle. A particular interval cycle is one that is a member of a specific rotation class [1-6] ~ [6-1], and stratum class [1-6] ~ [1-6-1-6] ~ [1-6-1-6-1-6].

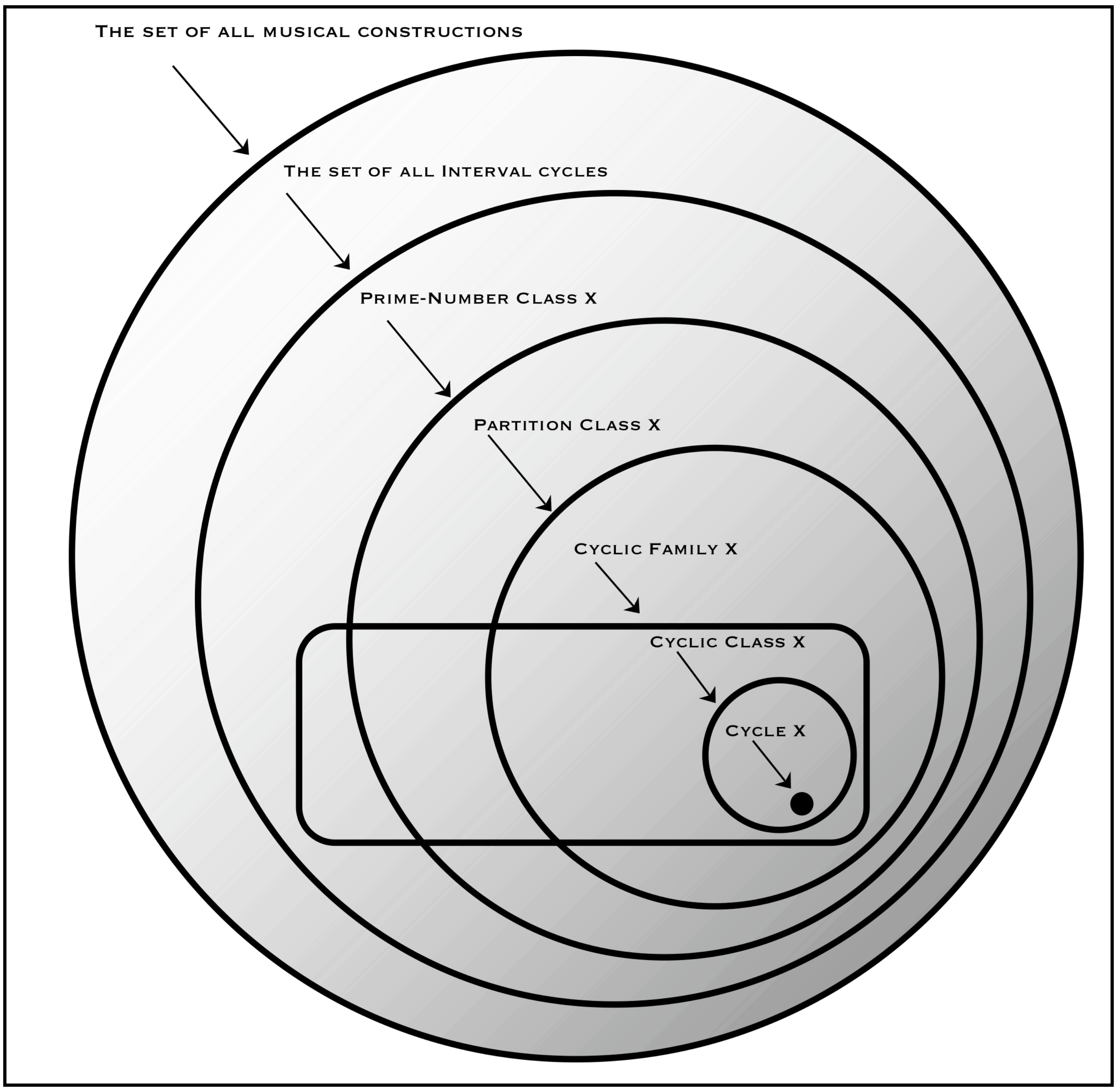


Figure 4.6. Equivalence relations.

Further equivalence relations not illustrated in Figure 4.6 could certainly be implemented into a composition whether they be logical or subjective in nature. For instance, one could impose as invariant a specific *cardinality class* regardless of partition, whereby the cyclic class 5-4 ~ 7-4 ~ 8-4 ~ 11-4, and so forth. Similarly, a specific prime-number class could be imposed on cardinalities rather than partitions, whereby cyclic class 5-2 ~ 5-4 ~ 11-8 ~ 21-16, and so forth. More subjectively, one could impose as invariant that the music be played by a kazoo orchestra. Clearly, the possibilities are endless.

These equivalence relations can be differentiated from those found in set theory. Octave and inversional equivalency, underlying the concepts of pitch class and interval class, are foreign concepts in the cyclic method. For instance, whereas inversional equivalency means that intervals 5, 7, 17, 19, 29, 31, 41 and 43 are simply instantiations of interval-class 5, the cyclic method places them into completely separate cyclic families. Underlying the concept of cyclic families (and deliberate departures from them by way of multilevel invariance), are voice-leading and mapping relations capable of coalescing concomitant coherence in pitch space, thus unifying otherwise totally unrelated pitch-class sets. The cyclic method is a clear alternative to tonality or other forms of post-tonality, since it operates on many levels of embedded equivalence, any one of which in isolation would be sufficient for unifying a composition without recourse to tonality. At the same time, tonality is remanded within its purview as a highly localized instantiation.

CHAPTER V
ANALYSIS OF *SYMPHONY NO. 1*

This chapter explores how the compositional method, as outlined above, is applied to *Symphony No. 1*. It should be noted that this examination is limited to only a few exceptional or structurally important moments throughout the composition, as it would be impractical to do otherwise. The detail devoted to even these examples also varies depending upon importance and/or the extent to which a particular topic has been covered.

As the title suggests, *Symphony No. 1* is not programmatic. The lack of extramusical suggestion engenders the listener to adopt an aesthetic attitude appropriate for free association. It is believed that compositional choices should be dictated by the mandates of logic. Thus, a formalist viewpoint has been adopted whereby the ultralogical nature of the music is considered paramount. The composer's intentions are rendered immaterial to interpretation, while those of the listener are elevated.

Symphony No. 1 consists of three movements: I. *Adagio, Moderato, and Allegro,* II. *Lento*, and III. *Allegro.* Each movement makes use of a different set of prime-number classes: the first movement is assigned prime-number classes 7 and 19; the second movement is assigned PNC 2 and 11; and the third movement is assigned PNC 3, 5, 13, and 17. The movements are unified by the fact that every musical construction is an interval cycle. Figure 5.1 shows that they are also unified by intermovement interval-class relationships: 5, 7, 17, and 19 (movements 1 and 3) are instantiations of interval class 5. Similarly, 11 and 13 (movements 2 and 3) are instantiations of interval class 1. However, it should be noted that this interval-class comparison is a tenuous one, since prime-number classes also contain partitions that are multiples of the initial prime numbers. For instance, prime-number class 7 also contains 14, which is an instantiation of interval class 2. Nevertheless, an interval-class comparison actually reinforces the idea that the movements are intimately bound to one another.

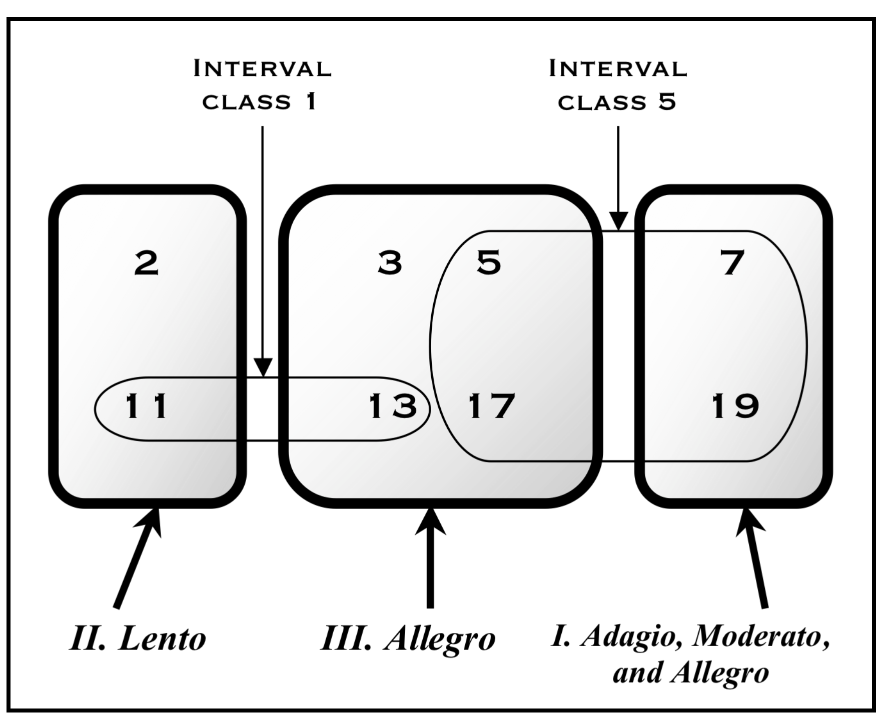


Figure 5.1. Prime-number class and interval-class relationships.

In the formal analysis of these movements, it should be noted that all movements are in a sectional, through-composed formal layout. No attempt was made to follow any previously established symphonic formal conventions, save for the general arrangement of movements into a fast-slow-fast pattern. Furthermore, formal divisions are not based upon melodic themes, but rather, upon cyclic classes and prime-number classes. That is not to say that other considerations are impertinent. Even though largely dependent upon exclusive classes, indicated formal demarcations may encounter occasional encroachments from foreign classes. Hence, of necessity, formal boundaries take into consideration aspects of contour, dynamics, rhythm, character, and so forth. A graphical and formal outline of *Symphony No. 1* in its entirety can be found in Appendix A.

Indicated tempo markings are only recommendations to be deviated from at any moment upon the discretion/whim of

the conductor. The same statement applies to indicated dynamics and bowings, although to a lesser degree. This does not belie uncertainty on my part. Quite the contrary, it indicates an absolute certitude in the ability and necessity of conductors to be flexible. After all, deviation, variation, and adaptation are unquestionable hallmarks of a good symphonic performance.

Movement I: *Adagio, Moderato, and Allegro*

From a cyclic-class perspective (which parses the material more finely), the first movement consists of the following sectional, through-composed form: A^1, B^1, A^2, C, A^3, D^1, B^2, D^2, E, D^3, D^4 (Table 5.1 a). From a prime-number class perspective (which more coarsely parses the material), the form can be described as follows: A^1, A^2, A^3 (Table 5.1 b), where A consists of microform (a, b), such that *a* represents PNC 7 and *b* represents PNC 19. Although this latter method is more concise, it is less detailed, demarcating an imperceptible background construct. For the following analysis, the cyclic-class (middle ground) perspective is used.

Table 5.1. Formal Layout of Movement I: *Adagio, Moderato, and Allegro*.

a. Cyclic class perspective

Form:	**A^1**	**B^1**				**A^2**	**Bridge**		**C**	**A^3**
Prime-Number Class:	**7**									
Cyclic Class:	14-2	28-4	7-3	14-5	7-1	14-2	14-6	7-1	14-3	14-2
Measure Numbers:	1-15	15-18	19-28	29	30	31-34	35-37	38-39	40-47	48-52

Bridge	**D^1**		**B^2**		**Bridge**		**D^2**		
19		**7**				**19**			
19-4	19-2, 19-5, 19-3	14-2	7-3	7-1, 19-2	7-3	19-2	19-2	19-3, 19-4	19-4, 14-3
52-54	55-65	65-66	67-72	73	74-78	79	80-86	87-90	91

E			**Bridge**	**D^3**	**D^4**
7				**19**	
14-6	14-5	14-4	7-3	19-3	19-5
92-96	97-101	102	103-111	112-121	122-138

b. Prime-number class perspective

Macro Form:	**A^1**		**A^2**		**A^3**	
Micro Form:	**a^1**	**b^1**	**a^2**	**b^2**	**a^3**	**b^3**
Prime-Number Class:	7	19	7	19	7	19
Measure Numbers:	1-52	53-64	65-78	79-91	92-111	112-138
Number of Measures:	52	12	14	13	20	27
	64		27		47	

The A^1 section of the first movement (mm. 1-15), orchestrated exclusively for strings, begins at a slow tempo (♩ = 40) with a six-measure static cycle [7] with staggered entrances going from outer voices to inner voices. Figure 5.3 shows a texturally reduced excerpt of measures 7-11. In mm. 7-8, a four-voice texture states a two-measure metacycle (indicated by large bracket above the staff), consisting of cc 14-2 cycles excluding only [1-13] and [2-12]. The metacycle is then restated (mm. 9-10) in a five-voice texture and in rotation 1, followed by a return to rotation 0 (mm. 11-12) in a six-voice texture. Clearly, the metacycle is optimal since the number of rotationally distinct statements equals the cardinality of the cycles. As the texture thickens, the violas, cellos, and contrabasses are divided into two parts each. In both restatements, rhythmic diminution occurs, requiring intermittent remeterings to 7/8. Note the cyclically adjacent voice pairs with starting pitches in m. 7 (C, D) and (G, A), which move in parallel motion by the partition interval 14. Consecutive metacycles overlap by at axis points.

Note the melodic character of the voice leading. Melodiousness is facilitated by widely-spaced voices making for easily

discernable independent lines, frequent use of directionally opposed steps preceding and following leaps, and use of sustained and contrary motion ensuring a gently rising slope as opposed to one that is steep, resulting in adjacent metacycles that are separated by only 4 semitones (which is remarkable considering their length).

a = [7-7], b = [6-8], c = [4-10], d = [5-9], e = [3-11]

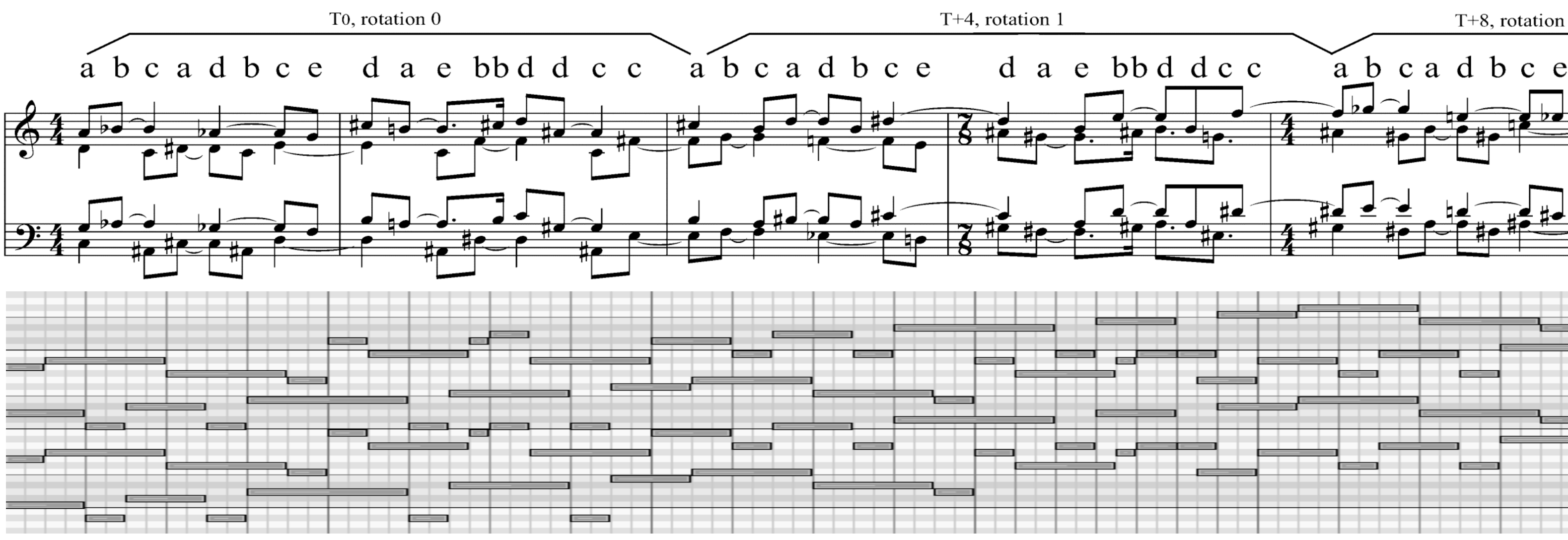


Figure 5.2. Excerpt of Movement I (mm. 7-11) consisting of metacycles from cc 14-2.

In measures 13-15, cycle [7] moves in parallel motion to a cadence on another cycle [7] as the tempo increases (♩. = 46) and the meter changes to 6/8 to conclude the A[1] section. In measure 15, a sforzando clarinet in altissimo register heralds the B[1] section (mm. 15-30), while strings briefly interrupt (mm. 16-17) with horizontal statements of cycle [1-2-1-3] (cc 7-4), which form vertical cycles from cyclic class 28-4. The strings reluctantly submit to the woodwinds as the tempo increases (dotted quarter = 46) and the meter changes to 6/8. This section consists of repetitions of a two-measure metacycle of cycles from the tightly packed cyclic class 7-3. The distance between adjacent voices in these cycles must be necessarily small, the resulting clustered effect of which contrasts with the uncompressed cycles of the A[1] section. As Figure 5.3 shows, this is an optimal metacycle, since three statements of this two-measure pattern (indicated in brackets) occur before transposition, which is equal to the cardinality of the cycles. The number of possible rotations is likewise equal to the cardinality.

a = [1-3-3], b = [2-2-3], c = [2-1-4], d = [1-2-4]

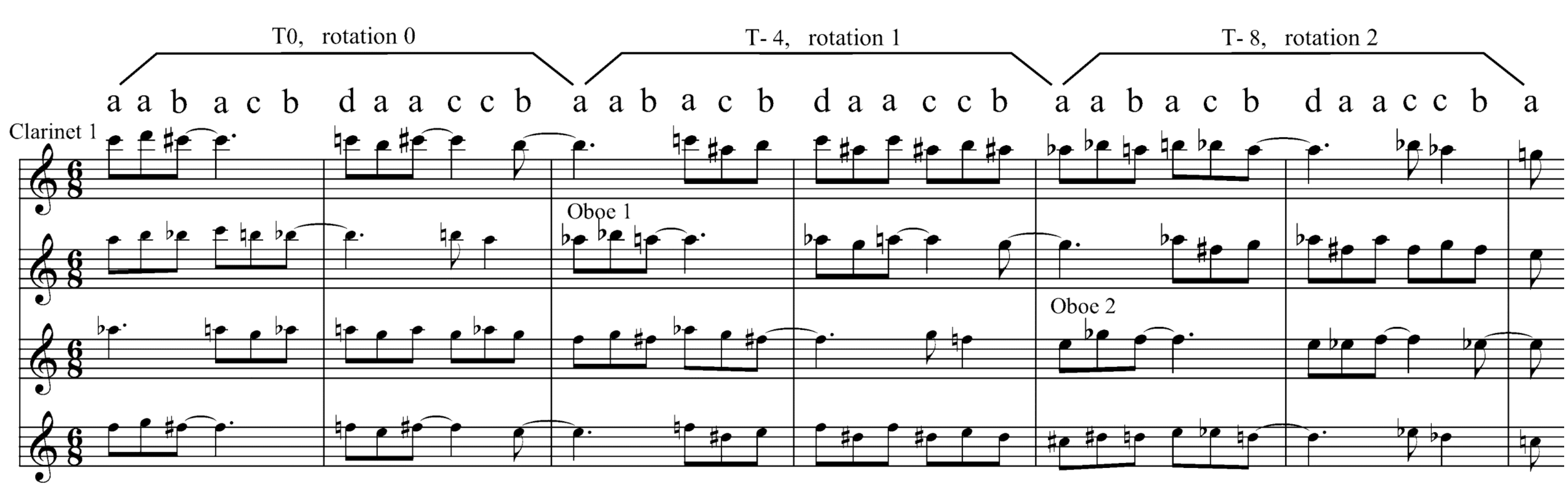


Figure 5.3. Excerpt from Movement I (mm. 16-22) containing metacycles from cc 7-3.

The texture gradually thickens from one voice (mm. 16-18) to nine voices (mm. 27-30), not including the accompanimental pedal [7] cycles in the strings and low brass. The B[1] section cadences on a cycle [7] chord (m. 30). The continued return of cycle [7] acts both as a point of repose and as a reminder of the prime-number class 7, of which many of the cycles in this movement are members.

Due to its reduced texture (four-voices) and descending contour, the brief A[2] section (mm. 31-34) acts as a release from the tumultuous clusters of section B[1]. It is orchestrated for the rich combination of clarinets and horns, and shares the cyclic class 14-2 in common with section A[1], as well as its concomitant wide spacing. However, the metacycle is different. As Figure 5.4 shows, a one-measure metacycle in 7/8 meter consisting exclusively of [5-9] cycles is stated twice (mm.31-32). Like many passages in *Symphony No. 1*, what is special about these measures is their exploration of voice-leading possibilities in the face of a reiterated cycle type. Next, a two-beat metacycle in 4/4 meter is stated twice (m. 33) made up of cycles [5-9] and [4-10], followed by a one measure pattern (m. 34). Nonfunctional pedal tones are sustained throughout this section in the violins and contrabasses, which are terminated in converging glissandi (mm. 34-35). Even though this section is comprised of multiple metacycles, voice leading remains smooth throughout, with every adjacent metacycle sharing a common tone. This is quite typical throughout *Symphony No. 1*.

a = [5-9], b = [4-10], c = [3-11], d = [7-7]

Figure 5.4. Excerpt from Movement I (mm. 31-34) showing metacycles from cc 14-2.

A short bridge begins with a two-measure scalar passage of cycle [1-3-1-3-3-3] (cc 14-6) in measures 35-36 (Figure 5.5). The cycle remains harmonically static, acting as a source of scalar material for 13 voices – a large increase in textural mass – which move in pervasively contrary motion outwards to measure 37. The passage is orchestrated for spiccato violins and cellos ( ); woodwinds, violins, and contrabasses ( ) and leaping trombones ( ) Clearly, there is no need for strict rhythmic alignment when cycles are utilized primarily as scales, yet it is advisable to be attentive to resulting verticalizations, which in this instance are widely spaced, in spite of the tightly-packed source material.

The thickened texture and rhythmic complexity are usurped in measure 37 by sparsely-voiced [7] cycles in the outer voices, which move in free contrary motion followed by an eight-voice [7] cycle moving in parallel motion (mm. 38-39).

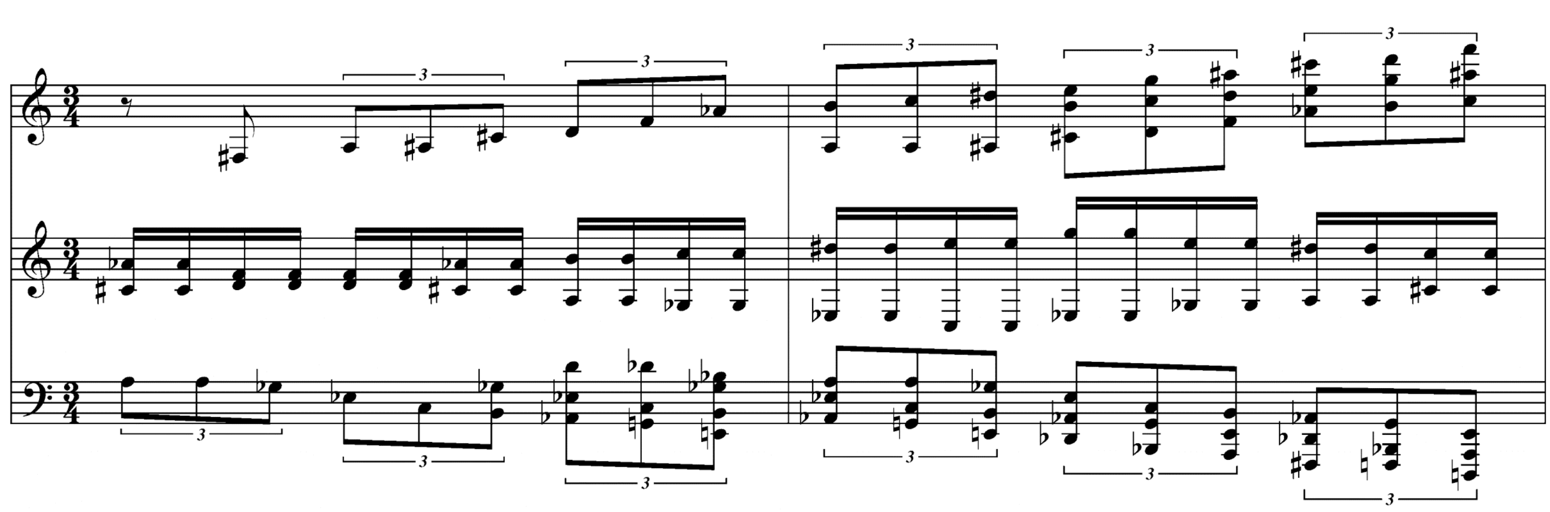


Figure 5.5. Excerpt from Movement I (mm. 35-36) illustrating a scalar traversal of a [1-3-1-3-3-3] cycle.

Section C (mm. 40-47) consists entirely of cyclic class 14-3, and begins with an abruptly thickened texture of ten independent voices orchestrated for the full brass section and supporting woodwinds. The tempo is also abruptly increased to ♩. = 64.

Beginning in a 6/8 meter, a cyclic class 14-3 metacycle is stated in measures 40-41, and a second time (rotated) in measures 42-43 (beginning on the la of beat one). In measures 44-45, a metacycle consisting exclusively of [3-3-8] cycles, forms two successive palindromes, first ascending, and then descending (Figure 5.6). The arrows indicate the peak and valley (axes) of the two palindromes. This moment of harmonic invariance is contrasted by a simultaneous rhythmic divergence in which the harmonic rhythm changes from eighths (6/8 meter in measures 44-47) to triplet quarters (4/4 meter in measures 44-45). The tempo does not change (eighth = eighth) so the audible result is a sudden deceleration.

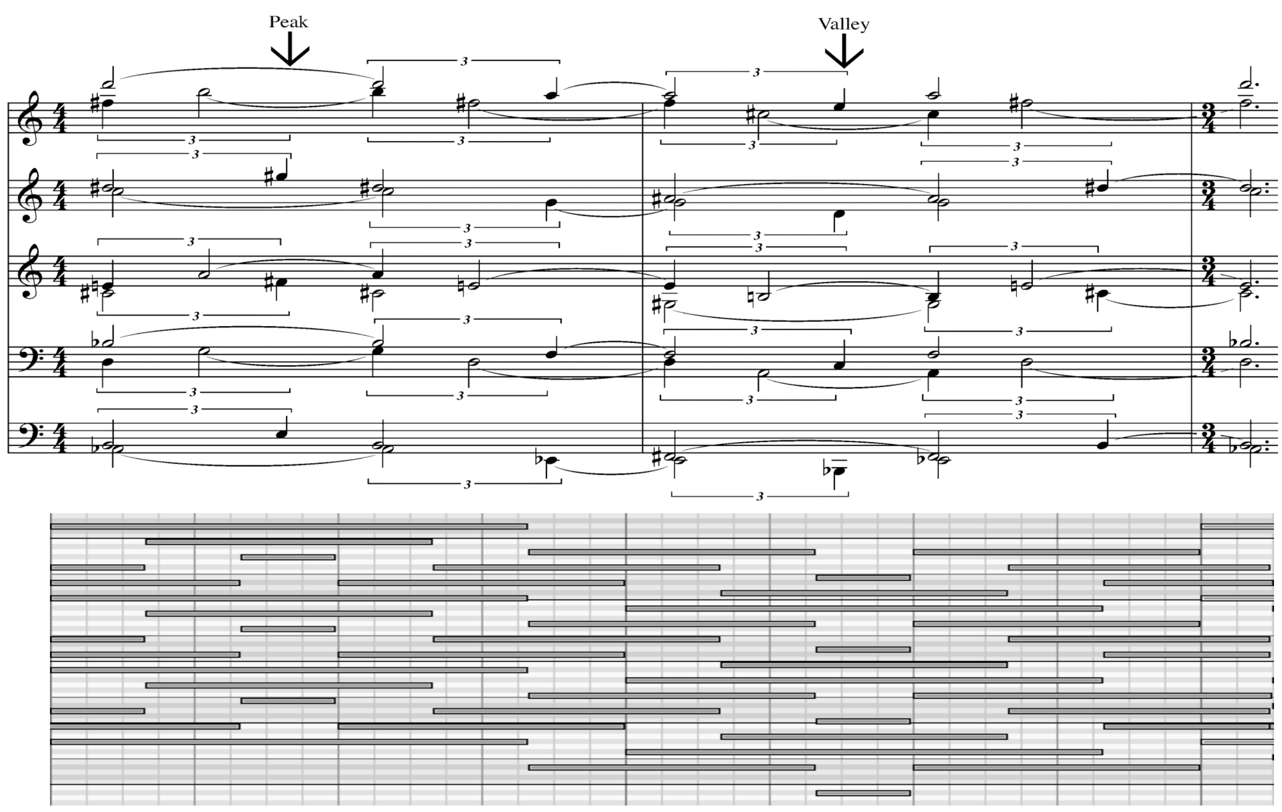
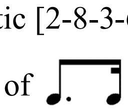


Figure 5.6. Excerpt from Movement I (mm. 44-46) consisting of [3-3-8] cycles.

There is a simultaneous textural divergence at this point in the form of a climax by way of the first full orchestral tutti of the composition (mm. 44-48). This is achieved by unison doubling while the underling harmony of ten voices remains unchanged. The horizontal motion of the voices in this section is characterized by perfect fifth leaps, some of which are filled in by trombone and string glissandos. It ends with ascending scalar statements of cycle [1-2-1-3] (similar to mm. 16-17) leading to a resolution on cycle [4-5-5] and pick-ups to the next section on beat three in the form of a [3-11] cycle.

In section A^3 (mm. 48-52), the texture is reduced to eight voices and the distances between voices widen to fill out cc 14-2 cycles. From measures 48-50, the voices move in a descending contour as an eighth-note harmonic rhythm alternates between cycle [5-9] and cycle [3-11]. In measure 51, an alternation between [4-10] and [5-9] is ultimately resolved by means of a cadence on [5-9] (m. 52). Another bridge section (mm. 52-54) presents a static [2-8-3-6] cycle (interval class 19-4), from which horizontal lines are derived. These horizontal statements consist primarily of  ascending leaps with simultaneous  descents. This scalar passage marks the first appearance of prime-number class 19.

Section D^1 (mm. 55-66), is characterized by cycles from prime-number class 19, as are all D sections. This section is an exercise in textural mass variation via cyclic-class modulation. The orchestration is mixed: some doublings occur and all instrumental families contribute, while none stand out. Like section A^3, this section begins at a dynamic and registral peak with six voices outlining widely spaced cc 19-2 cycles in a gradually descending horizontal contour and diminuendo (m. 55). In measure 56, the texture rather abruptly increases to cc 19-4, which are essentially thinned-out [4-2-4-4-5] cycles (cc 19-5). As Figure 5.7 shows, these [4-2-4-4-5] cycles are eventually filled out in (mm. 57-59) in the form of short metacycles as ten voices continue their descent. Although only consisting of two interval cycles, this metacycle is clearly optimal (five statements = cardinality 5).

a = [4-2-4-4-5]

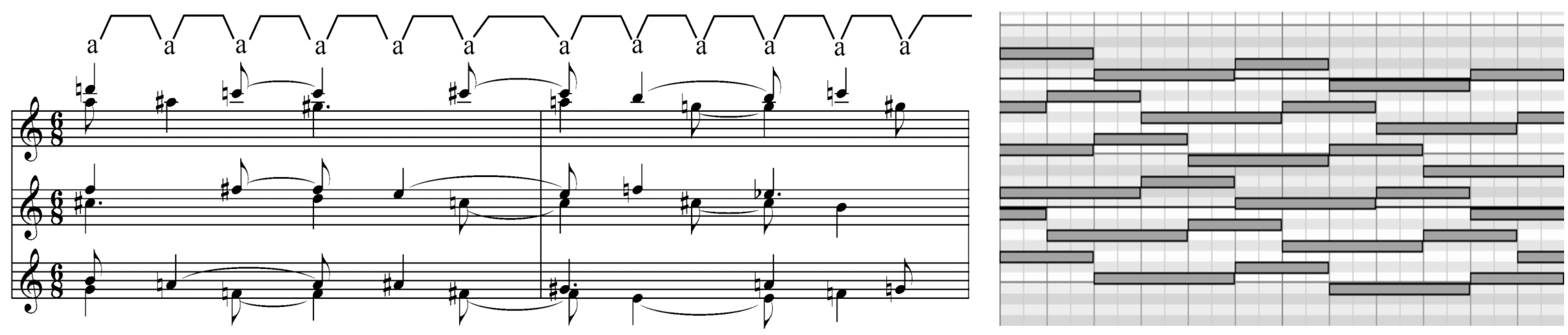


Figure 5.7. Excerpt from Movement I (mm. 57-58) composed of [4-2-4-4-5] cycles.

In measures 60-65, the texture is reduced to six voices, which outline cyclic class 19-3, and beginning in measure 62, the contour changes to a ascending motion with crescendo. In measures 61-65, the rhythm changes to quarter-note triplets traversing [3-8-8] cycles forming palindromic metacycles featuring melodic leaps by perfect fifth (Figure 5.8). All of this is remarkably similar to measure 44-45, but the interval cycles used are completely different: [3-3-8] versus [3-8-8]. Although the interval vectors of the two cycles (per single stacking) are not identical, [112101] versus [101310], the similarities of the cycles and passages cannot be denied. This is more a function of the voice-leading distance for individual voices than anything else, a situation ensuing from similar, if not identical, interval content.

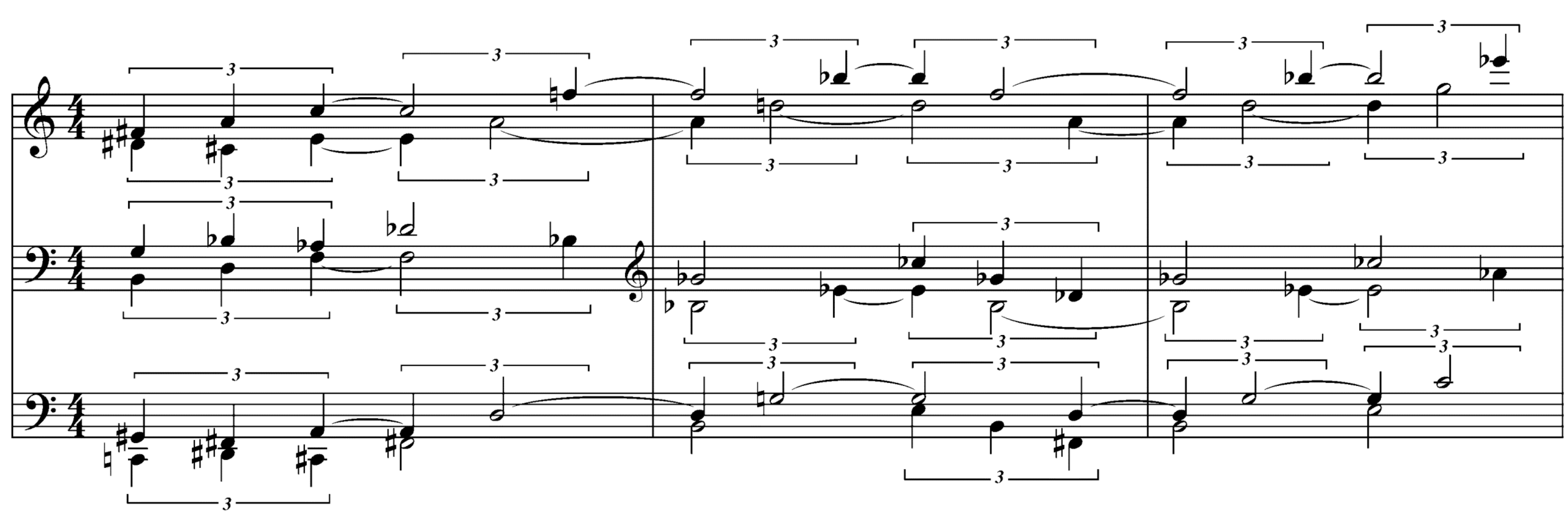


Figure 5.8. Excerpt from Movement I (mm. 62-64) comprised of [3-8-8] cycles.

Two parallel [5-9] cycles on the tail end of measure 65 arrive at a by now familiar cadence on cycle [7] in measure 66 functioning both to close out section D^1 and to dovetail with the next, since the prime-number class here has changed to 7.

The B^2 section (mm. 67-73) marks a return to prime-number class 7. It begins with a one-measure optimal metacycle of alternating [2-2-3] and [1-3-3] cycles (cc 7-3). As Figure 5.9 shows, this section is an exploration of variable voice-leading despite limited cycle choices. In effect, the question is asked "how many ways can one go from point a to point b?" This question may, in fact, be the fundamental question that the entire symphony seeks to answer.

a = [2-2-3], b = [1-3-3]

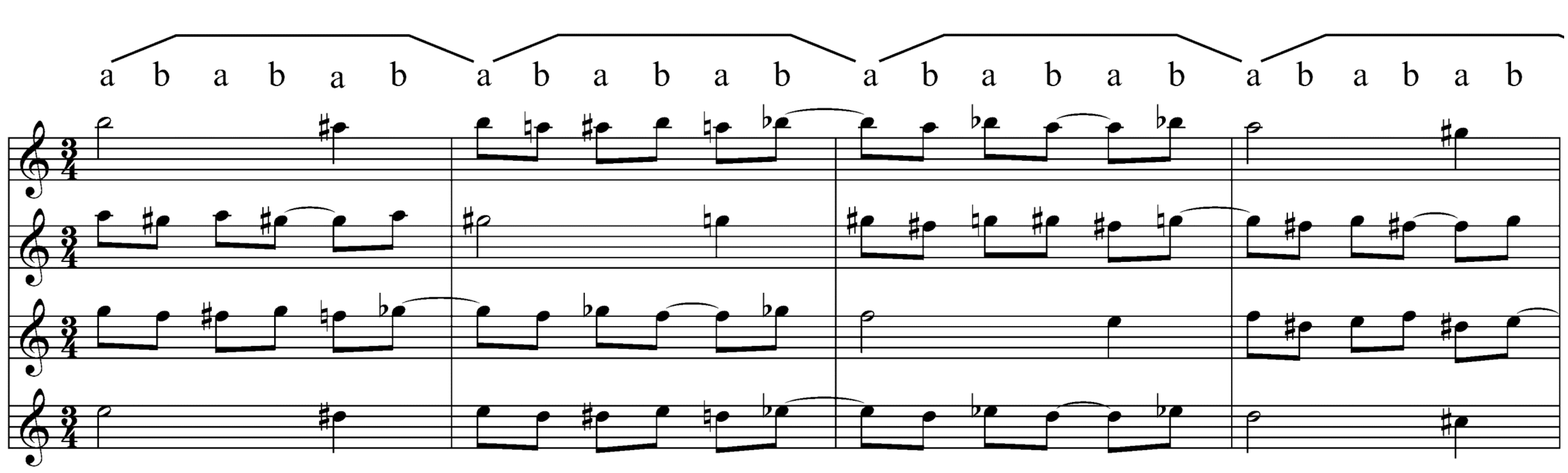


Figure 5.9. Excerpt from Movement I (mm. 67-70) consisting of alternating [2-2-3] and [1-3-3] cycles.

The tempo increases for the final time in measure 74 to ♩ = 105, which will be maintained until the end of the movement. At this point a chaotic bridge section ensues (mm. 74-79) featuring a scalar treatment of cycle [2-2-3]. An excerpt of this is shown in Figure 5.10. Again note the rhythmic freedom allowed by this technique.

Figure 5.10. Excerpt from Movement I (mm. 74-78) showing scalar traversal of cycle [2-2-3].

Section D^2 (mm. 80-91) again returns to prime-number class 19. As Figure 5.11 shows, this section begins with a five-measure metacycle (mm. 80-84) and incomplete repetition (mm. 85-86) using cyclic class 19-2. The cyclic class used here is 19-2, thus the intervals between adjacent voices are very large, making for an uncompressed sonority advantageous to the perception and clarity of both outer and inner voices. This complexity and length of this metacycle is commensurate with the first movement's overall aesthetic. This particular complexity is born out of a process that is less mechanical than simpler metacycles. It involved the selection of each cycle and each voice leading on the basis of purely melodic aims of the composer. Since the metacycle is rotated, the inner voice later becomes the soprano voice (mm. 85-86), thus the selection process had to take this into account. While this primarily melodic oriented selection process can be seen at similar points in the second and third movements, it contrasts with much of the symphony's unabashedly harmonic orientation.

a = [9-10], b = [6-13], c = [8-11]

a a a b c c c c a a b c a c c c c a b c a b a a b b c c a a b b c c c a a b b c c a a a a a b c

Figure 5.11. Excerpt from Movement I (mm. 80-85) comprised of a metacycle from cc 19-2.

The remainder of the D^2 section (mm. 87-91) involves thicker cycles (cc 19-4). Figure 5.12 shows a series of metacycles consisting exclusively of [3-6-4-6] cycles (cc 19-4). As shown in Figure 5.12, this series causes canonic imitation due to the fact that absolute voice distance traveled per LCM, per metacycle, from the bottom voice up is (3 + 1) + (3 + 3) + (2 + 1) + (2+1+2+1) = 19, which is equal to the partition of the cycles used (19). This voice motion is indicated in the first measure/metacycle of Figure 5.12[18]. Note that this metacycle is optimal, since four statements are needed before a return to rotation 0, which is equal to the cardinality of the cycle [3-6-4-6]. When a passage is canonical, each successive metacycle may actually be considered to be the same transposition with only a change in its rotation.

[18] Note that the top voice is not counted since it is cyclically adjacent to the bottom voice, thus is considered to be a member of the next stacking per lcm.

Melodically, the voices outline the cycle [3-3-3-1-2-1-2-1-2-1], which besides being a partition of 19 has a pitch-class content equivalent to the octatonic scale. Measures 90-91, in ascending contour, are essentially a shortened retrograde version of the previous passage measure 87-89.

a = [3-6-4-6]

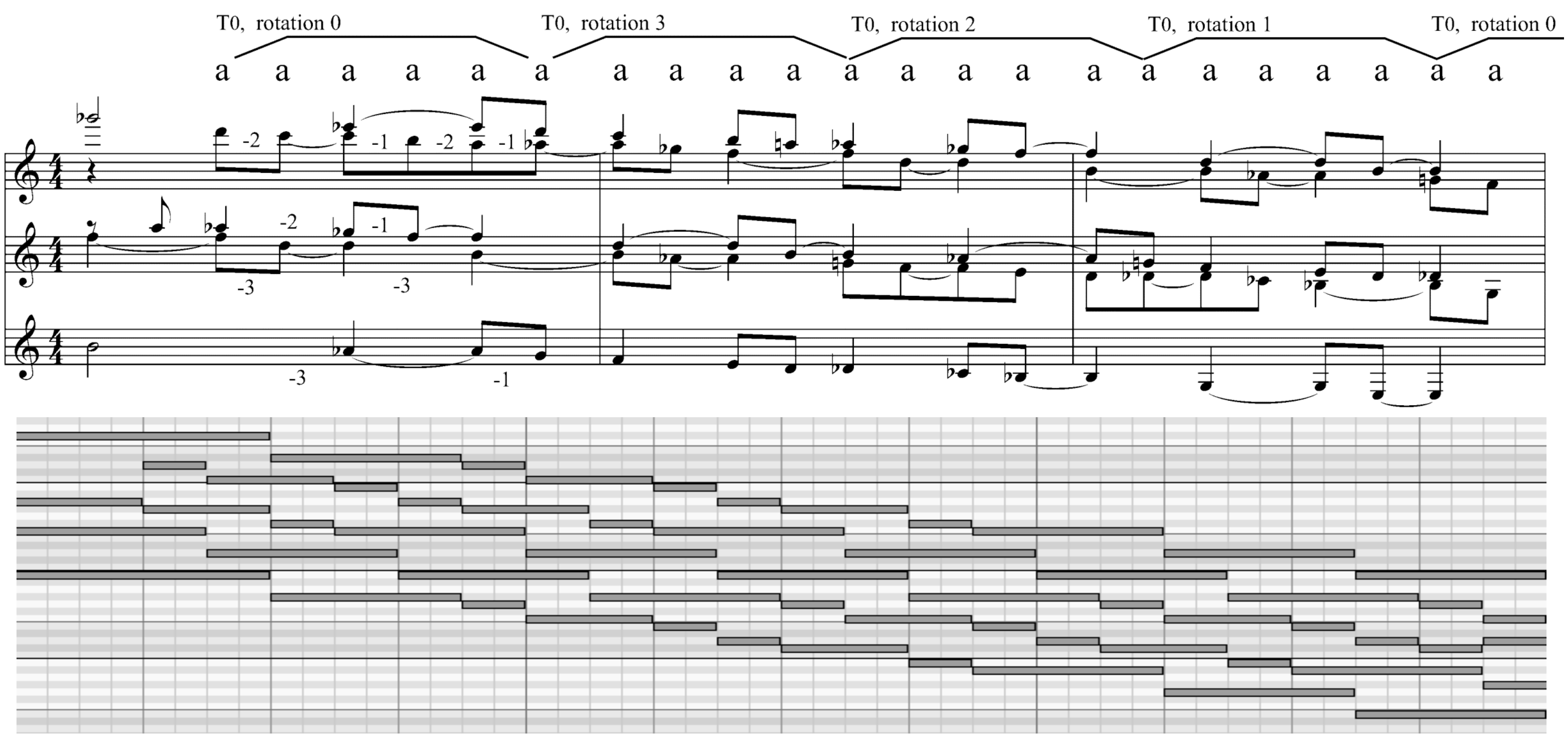


Figure 5.12. Excerpt from Movement I (mm. 87-89): canonic metacycle of [3-6-4-6] cycles.

Next, section E (mm. 92-102) enters, marking a return to prime-number class 7. However, the listener would have an extremely difficult time recognizing this fact, since this section begins with dense cyclic class 14-6 clusters, which until now have been absent. The cycles chosen are rich in semitones, which ratchets up the tension in preparation for the conclusion. The metacycle demarcated in Figure 5.13 occurs approximately three and a half times from measures 92-96. Note the random placement of sixteenths in place of eighths, which adds a bit of uncertainty to this passage.

a = [1-3-2-3-1-4], b = [1-3-1-4-1-4], c = [1-3-1-3-3-3]

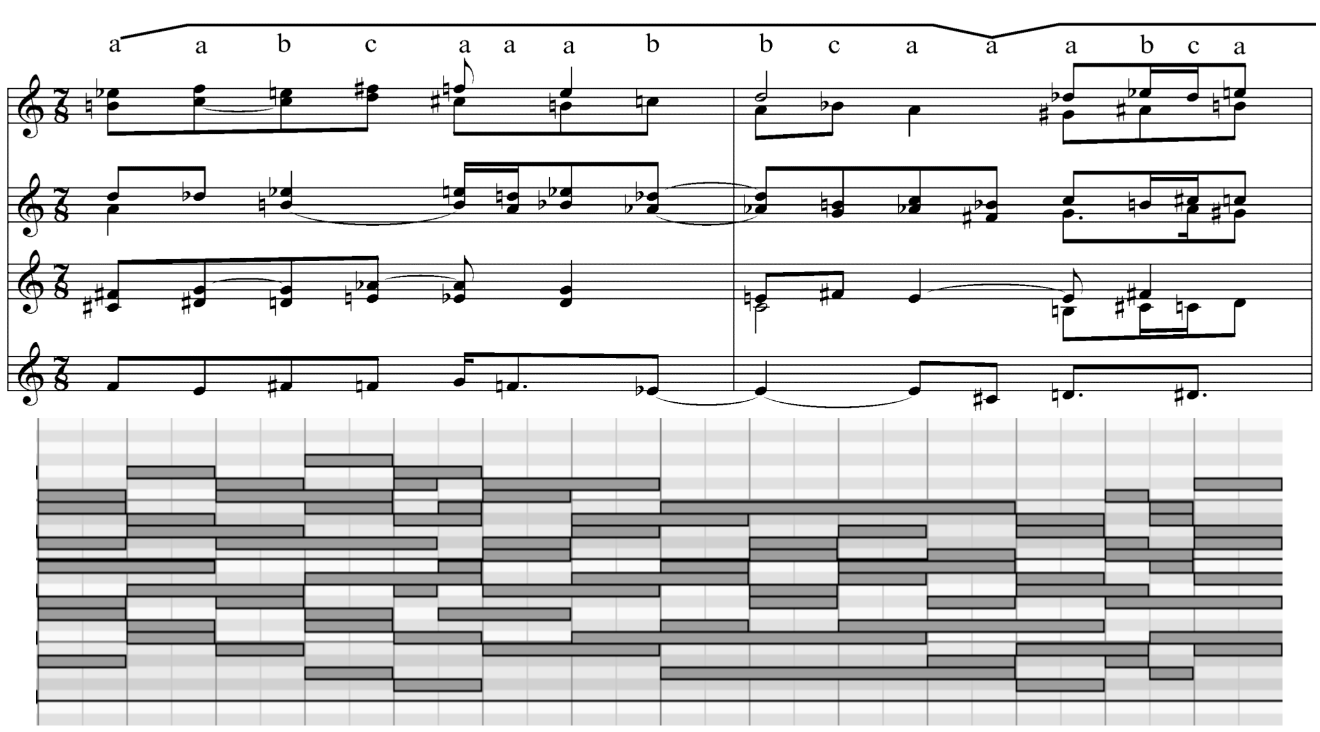


Figure 5.13. Excerpt from Movement I (mm. 92-93) showing a metacycle from cc 14-6.

In measures 97-102, the texture thins slightly to cc 14-5. The beginning of this passage (shown in Figure 5.14) lacks a well-defined metacycle upon which to unify the [1-4-1-4-4] cycles. This succession ascends to a peak on F#. On beat three of measure 98, an optimal metacycle gives form to the descending contour. The descent continues in measure 102 where the texture is again thinned to cc 14-4. The voice motion throughout is primarily by half and whole steps, minor third skips, and common tones.

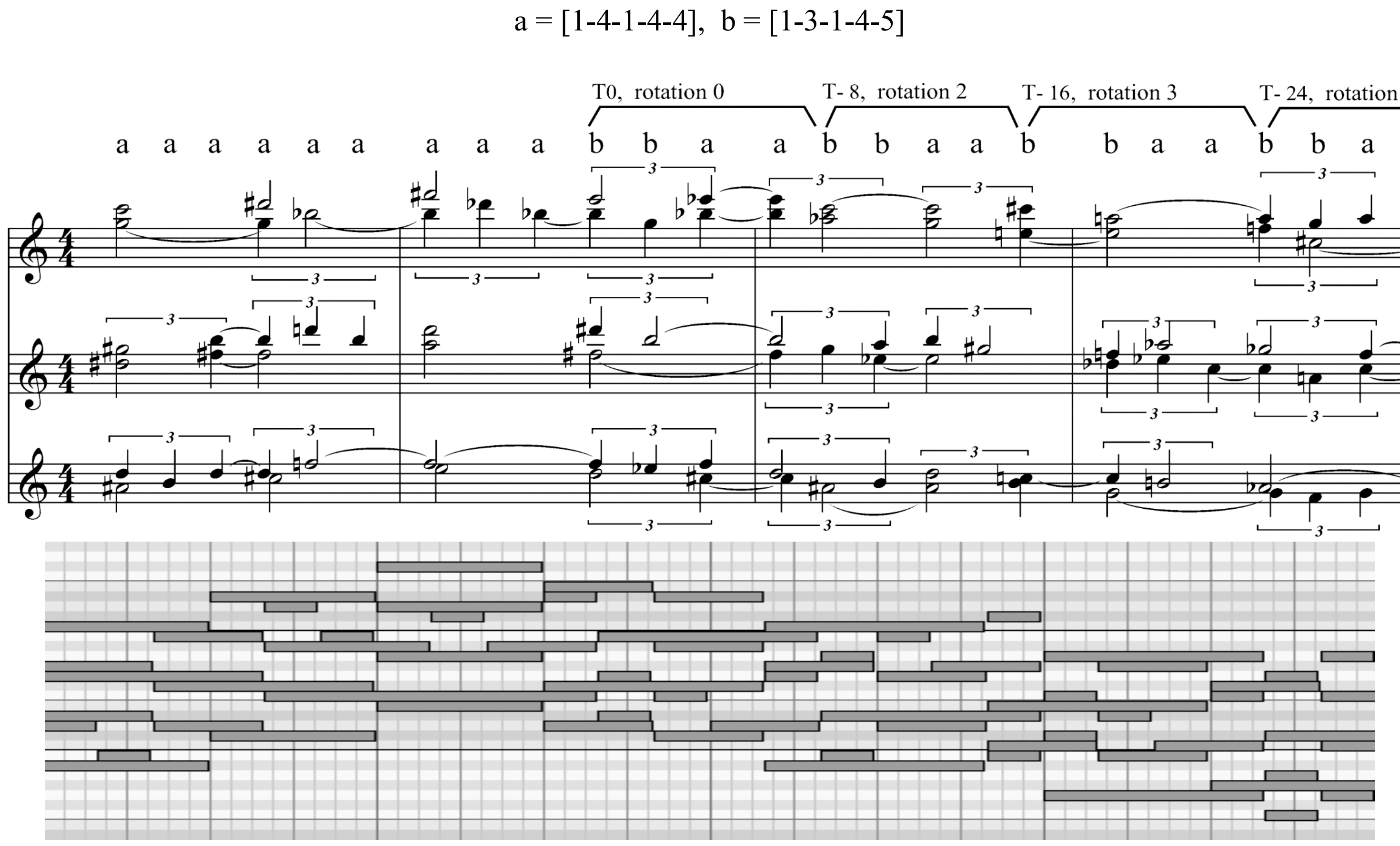


Figure 5.14. Excerpt from Movement I (mm. 97-100) illustrating metacycles from cc 14-5.

Another bridge occurs in measures 103-111. It consists of a scalar treatment of cycle [1-3-3] (cc 7-3) with a gradually ascending contour (see Figure 5.15). The vertical alignment of this scale forms an alternation between two distinct [3-3-4-4] cycles (cc 14-4). The alternation first occurs every quarter note (mm. 103-106), then every dotted quarter (mm. 107-108), and again every quarter (mm. 109-110). Using cycles as scales allows for greater rhythmic freedom, but in this case, the harmonic results are also controlled, which differentiates this from other scalar passages. What is remarkable is that both [3-3-4-4] cycles share common tones derived from a [7] cycle starting on the low G#: (G#, D#, A#, F, C). These common tones allow rhythms to be variable without sacrificing the integrity of the alternating vertical cycles. The rhythmic freedom afforded by horizontal cycles used as referential material is ideal for bridges, where forward momentum can be obscured. The overall effect of this passage is to maintain a static yet pulsating construction that builds upon the mounting tension before the conclusion.

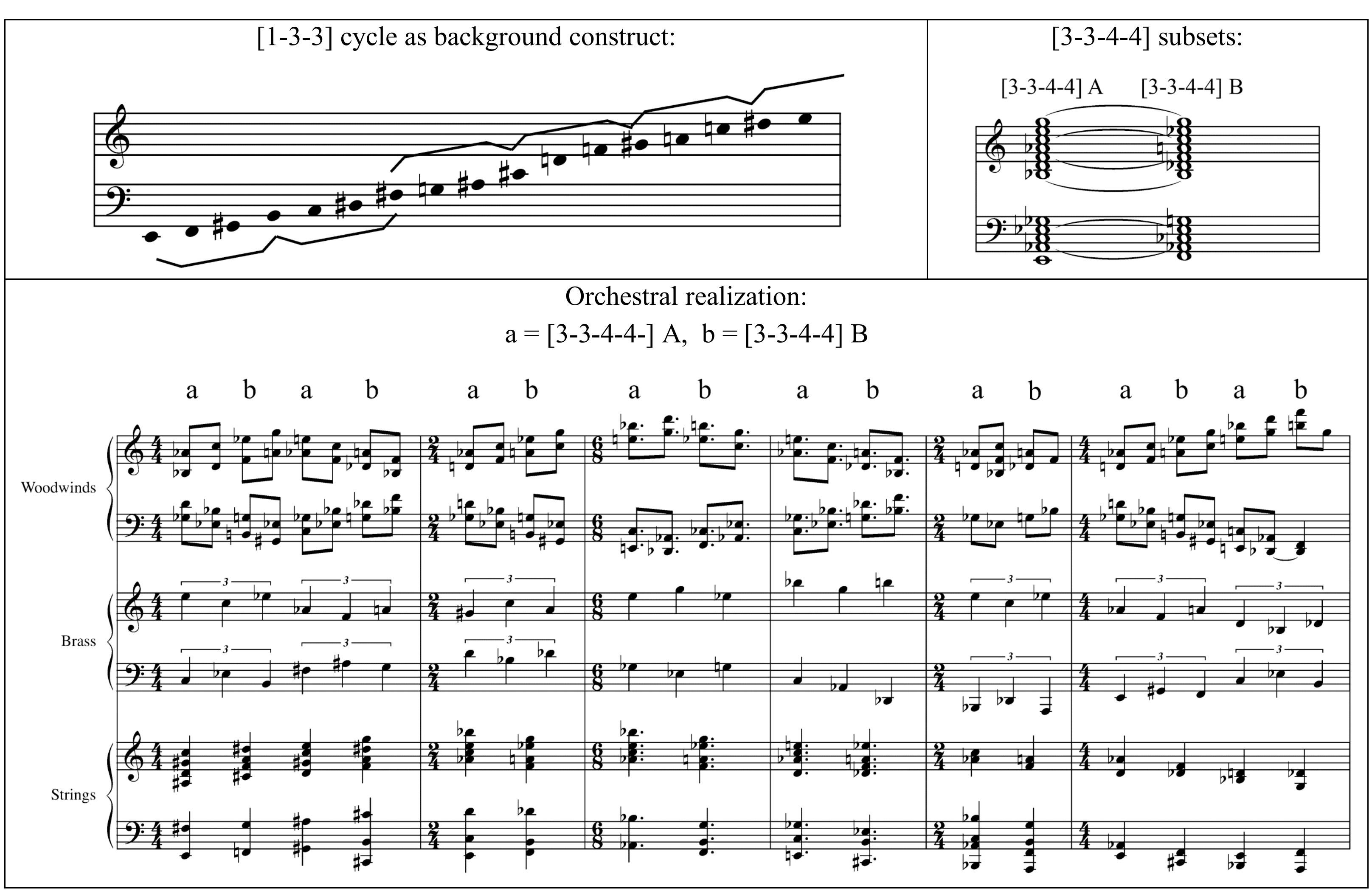


Figure 5.15. Horizontal [1-3-3] cycle alternating between two vertical [3-3-4-4] cycles (mm 105-110).

Section D^3 (mm. 112-121) is essentially a modified restatement of measures 61-65 consisting exclusively of [3-8-8] cycles (see Figure 5.8). The most notable difference is the augmented rhythmic values that draw out a laboriously ascending contour and prolong the ever-escalating tension. The contour and dynamics climb to a cadence on a [3-8-8] cycle in measures 119-121.

In measure 121, trombone and string glissandi pronounce the arrival of the concluding D^4 section (mm. 122-128), which consists of cycles from cc 19-5, which were first heard briefly in measures 57-59 (within section D^1). In this instance, the texture is an eleven-voice harmony orchestrated for tutti orchestra vis-à-vis unison doublings of which only a single stacking is shown in Figure 5.16. The effect is a massive wall of timbrally rich audio. As Figure 5.16 shows, this section is constructed primarily from a metacycle of alternating cycles: [2-4-5-4-4] and [2-4-4-5-4]. It is clear that this metacycle is not optimal, since every repetition is in the same rotation. After an ascent of approximately two metacycles and a non-metacyclic peak in measure 125, approximately three retrograded metacycles descend (mm. 126-131), finally terminating on a sustained cadence on cycle [2-4-4-5-4] (mm. 132-138) to conclude the first movement. Note that the cyclically adjacent outer voices of this example outline two different horizontal whole-tone scales (a semitone apart), which are exchanged during the retrograded passage.

a = [2-4-5-4-4], b = [2-4-4-5-4], c = [2-4-4-4-5], d = [3-4-4-4-4], e = [2-5-4-4-4]

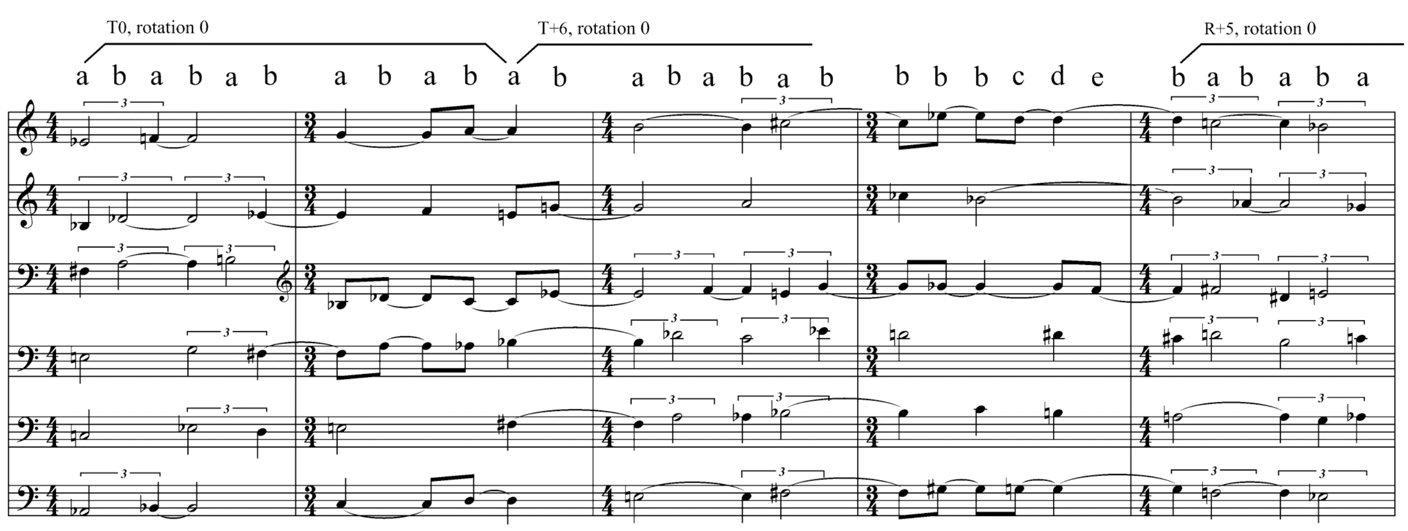


Figure 5.16. Excerpt from Movement I (mm. 122-126) comprised of metacycles from cc 19-5.

Movement II: *Lento*

Movement II, *Lento*, employs prime-number classes 2 and 11. Cycles from prime-number class 2 tend to be more pleasing in general, a quality that complements the intended mood of this slow middle movement (♩ = 50), yet is contrasted with cycles from PNC 11, which are generally more dissonant. The through-composed form is A^1, B, C, D, A^2, E, F (Table 5.2 a), but the background level (the prime-number class perspective) indicates a ternary form: A^1 B A^2 (Table 5.2 b).

Table 5.2. Formal Layout of Movement II: *Lento*.

a. Cyclic class perspective

Form:	**Introduction**	**A^1**					**B**		**C**	
Prime-Number Class:	2						11			
Cyclic Class:	16-5	32-7		16-2	32-7	16-2	11-6	11-2	11-6	11-2
Measure Numbers:	1-3	3-10	11-12	13-14	15-16	17-18	19-21	22	23-28	29

D			**A^2**			**E**		**F**
11	2							
11-4	8-3	16-5	32-7	16-2	32-7	16-3	16-2	16-2 and 16-4
30	31-33	33-34	35-36	37-38	39-41	42-48	48-50	51-59

b. Prime-number class perspective

Form:	**A^1**	**B**	**A^2**
Prime-Number Class:	2	11	2
Measure Numbers:	1-18	19-30	31-59
Number of Measures:	18	12	29

The introduction (mm. 1-10) consists of cycles from prime-number class 2. It begins with a three-beat metacycle of cc 16-5 cycles (mm.1-3). Every metacycle here is at a transposition distance of T-5, and a rotation of +1 from the previous one. Note that the horizontal lines here outline major and minor triads.

In measures 5-7, [2-5-4-5-2-7-7] cycles (cc 32-7) form two-beat metacycles. The same voice-leading path is reiterated every beat. Every fourth metacycle here is related by T+5, and every consecutive metacycle is rotated +2. In measures 8-10 the metacycle continues as before while rhythmic values continue to increase, first to and then to values, until a sustained cadence on cycle [2-5-4-5-2-7-7] in measures 9-10. Both of the metacycles in Figure 5.17 are optimal.

a = [3-2-3-2-6], b = [2-3-2-3-6], c = [2-5-4-5-2-7-7]

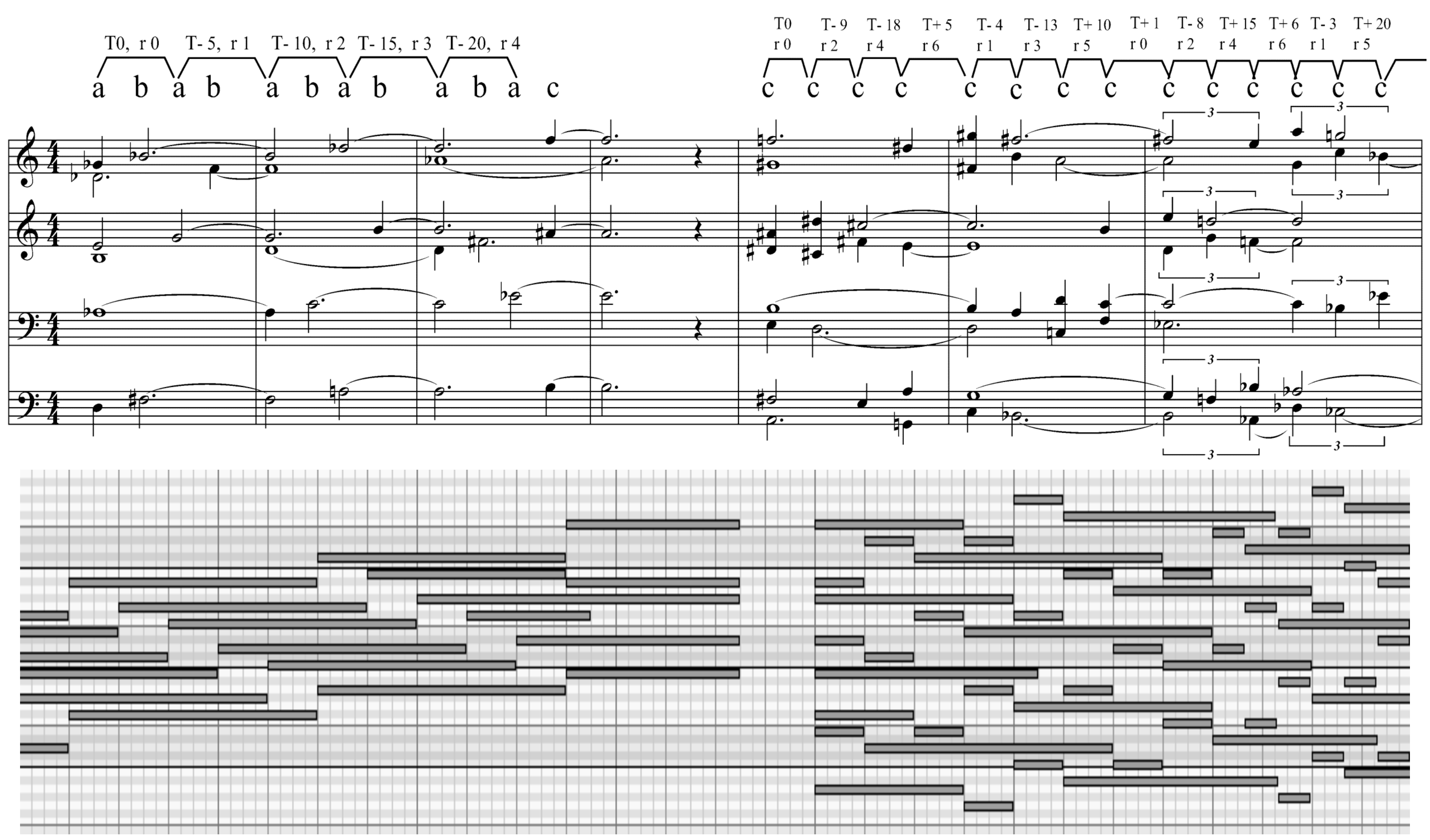


Figure 5.17. Excerpt from Movement II (mm. 1-7) showing metacycles from cc 16-5 and cc 32-7.

The A1 section (mm. 11-18) begins with palindromic metacycles comprised exclusively of the palindromic cycle [2-5-2-2-5-2-14] (cc 32-7). As we have already seen in several examples, when a musical passage maintains a single invariant cycle, the highest level of musical coherence is achieved, since this is most specific level (foreground) of equivalence classes (as indicated in Figure 4.6). Moreover, the voice leading is exactly duplicated from chord-to-chord, thus creating coherence on the most specific level of voice-leading path invariance. This is due to the fact that the metacycle here is the smallest possible as it consists of only two cycles. Again, this equates to one voice leading path reiterated every beat. Voice-leading here is a combination of similar motion (unidirectional) and common tone.

This metacycle is optimal, and all of the seven possible rotations are presented in order from 0 to 6. Note that the pattern backtracks after ascending to rotation 6, thus stops short of completion (rotation 0) at the axis. Remember that a metacycle is considered complete when the initial and terminal cycles are in the same rotation. Transposition occurs by consecutive whole tones [2] downward during the ascent, and in reverse during the descent. Note that only the outer voices are cyclically adjacent, since there is only a single stacking of the cycle here, thus countermanding the "two-stacking rule."

Statements of this metacycle can be found in the A1 section in mm. 11-12 and 15-16, as well as in the A2 section mm. 35-36 and 39-40. In each case they are bookended by a two-measure passage of alternating [5-11] and [3-13] cycles (cc 16-2) in measures 13-14, 17-18, and 37-38. Thwarting expectations, this construction does not occur in measures 41-42 as one would expect. Rather, measure 41 is filled with silence and measure 42 starts section E (see Figure 5.22).

a = [2-5-2-2-5-2-14]

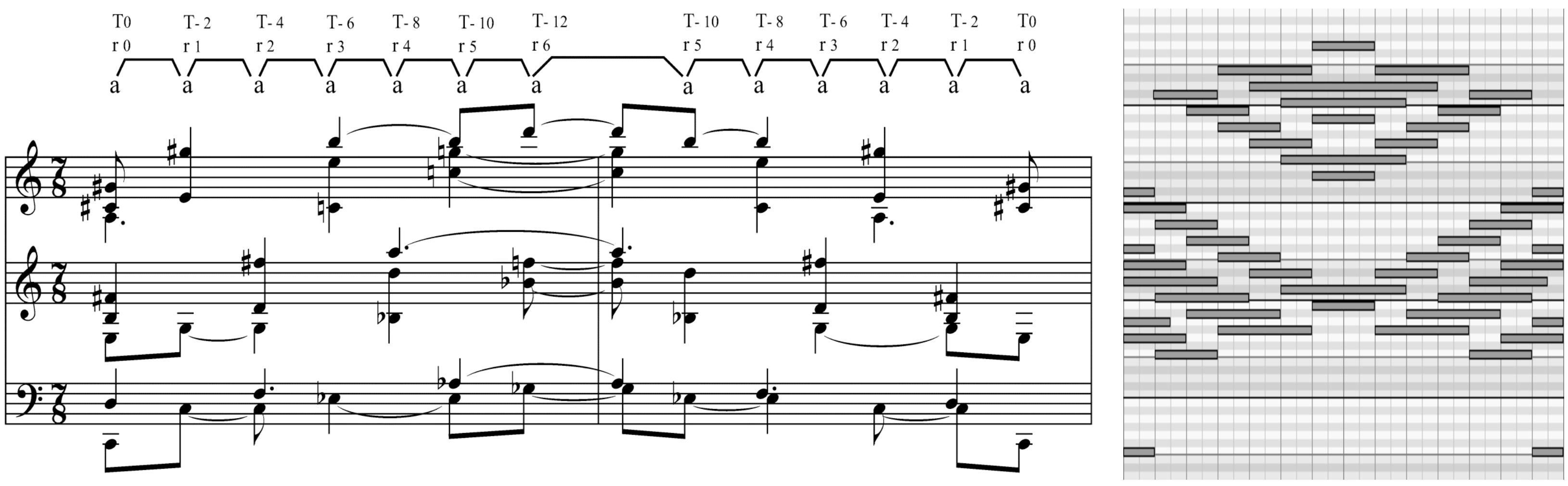


Figure 5.18. Palindromic arrangement of metacycles consisting of palindromic [2-5-2-2-5-2-14] cycles.

Section B (mm. 19-22) begins with alternating [1-2-2-2-1-3] and [1-1-1-3-2-3] cycles (cc 11-6) in measures 19-21. Consisting of twelve voices separated primarily by steps (semitones and whole tones), these are the most densely-packed cycles up to this point in Movement II.

The rising contour reaches a peak in measure 22 (shown in Figure 5.19), where a suddenly thinner texture and descending contour commences. This measure consists of metacycles of [3-8] and [4-7] cycles (cc 11-2). Interestingly, the horizontal movement consists of an upper voice moving by [3-5] and a lower voice moving by [4], both of which are from prime-number class 2, while the harmony remains solidly within prime-number class 11. In fact, this is quite a common occurrence, since only in the extraordinary cases of canonic imitation do the horizontal and vertical share a common PNC. Yet, as may have been deduced, horizontal movement is considered ancillary within the harmony-centric perspective of this document. Note that this metacycle is not optimal, since voice leading results in restatements that are not rotated (r 0). Also note that restatements of the metacycle always occur at the interval of a minor sixth [8] below the previous statement.

a = [3-8], b = [4-7]

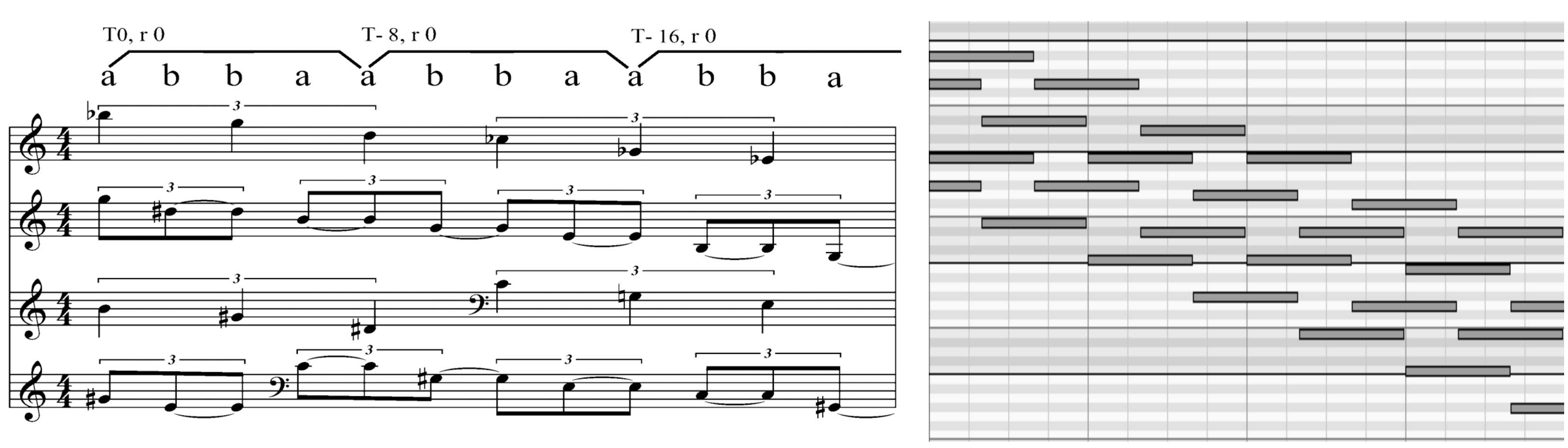


Figure 5.19. Excerpt from Movement II (m. 22) comprised of metacycles from cc 11-2.

Section C (mm. 23-29) begins with a three-voice texture that gradually increases as the tessitura rises. It is orchestrated primarily for strings. As Figure 5.20 shows, pitch content is derived from a single [1-1-1-3-2-3] cycle (cc 11-6) as a background construct (or scale). There is no specific metacycle dictating verticalizations, per se, rather they consist of subsets of the [1-1-1-3-2-3] cycle. Clearly, however, the passage is logically (if not systematically) arranged. Voices are widely spaced such that semitones are absent, and whole tones are rare, thus rendering impotent the full dissonant potential of the underlying referential construction. In fact, the horizontal lines, which are rhythmically paired with voices a perfect fourth apart, produce pleasing sonorities robustly imbued with exoticism – no doubt a resultant of the inherent potentialities of the [1-1-1-3-2-3] cycle. Moreover, by way of carefully placed contrary and sustained motion, the counterpoint facilitates the individualism of

these distinct voice pairs (of which there are five in total), a technique without which the texture would seem overcrowded. Section C ends with a cadence on [5-6] (cc 11-2), one of the possible subsets of [1-1-1-3-2-3].

Figure 5.20. Excerpt from Movement II (mm. 25-29) showing cycle [1-1-1-3-2-3] as a referential construction.

Section D (mm. 30-34), orchestrated primarily for brass, begins with cycles from cyclic class 11-4, thus sharing the same partition class as the previous section. However, this is maintained for only one measure (m. 30), as shown in Figure 5.21. What is unusual is that in measures 31 and 32, cycles from a completely different cyclic class (cc 8-3) enter. This modulation is carried out by way of common tones, and the entrance of an additional voice (second trumpet). Note that in measure 30, the first trumpet and fourth horn are cyclically adjacent by in the interval of 11. However, in mm. 31-32, the first trumpet is cyclically adjacent to the third horn by an interval of 8 (which is in-turn cyclically adjacent to the first trombone). The metacycles in measures 31 and 32 are optimal (rotated upon restatement) and ascend consecutively by a transposition of +5.

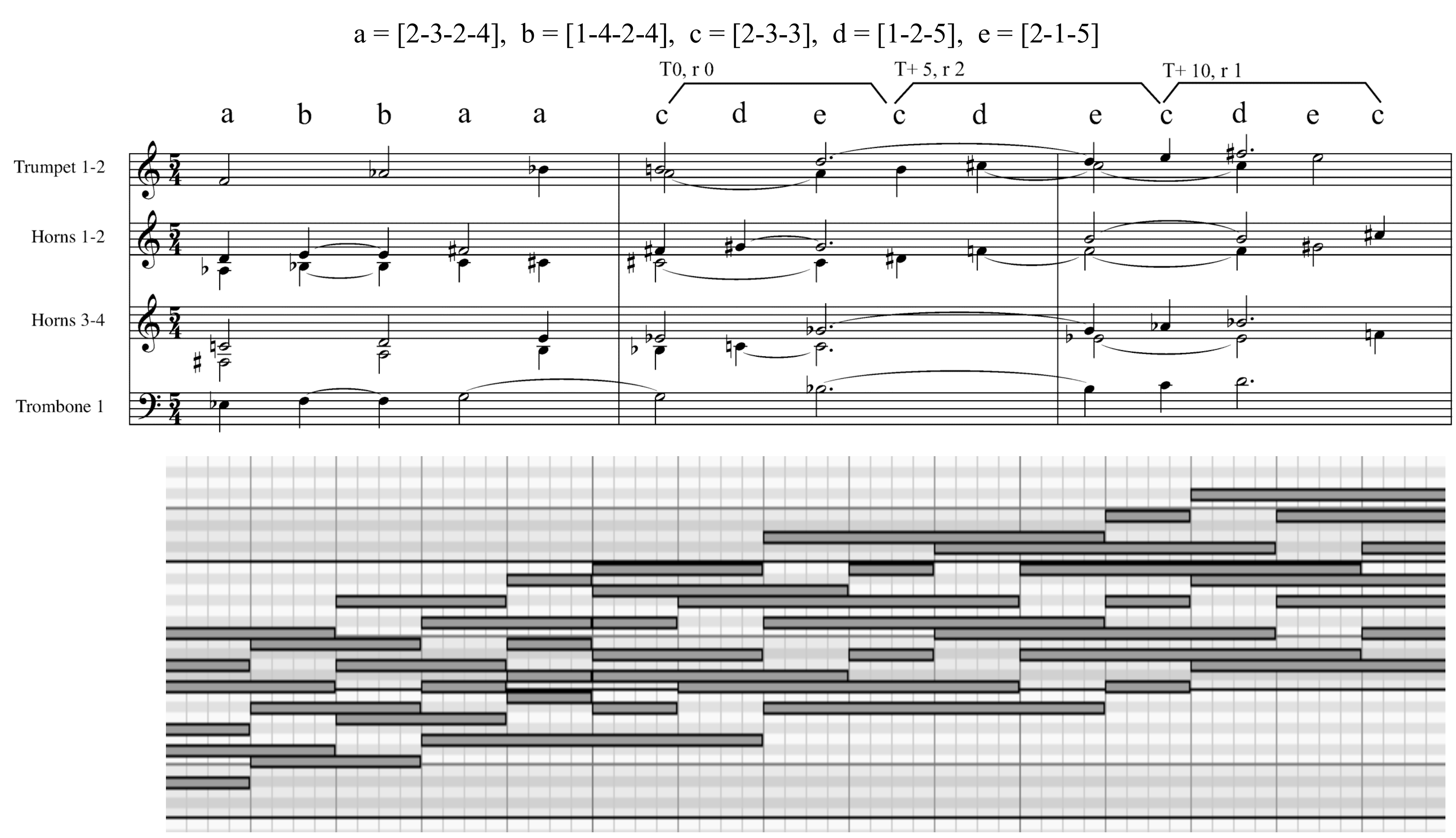


Figure 5.21. Excerpt from Movement II (mm. 30-32) illustrating a modulation from cc 11-4 to cc 8-3.

As already discussed in Figure 5.18, section A[2] (mm. 35-41) is a transposed and modified restatement of measures 11-18. Following a measure of dramatic silence (m. 41), the E section (mm. 42-50) commences. This section is made up of cycles from cyclic class 16-3. It is similar in aesthetic to Figure 5.11 from Movement I, in that a very long metacycle has been constructed out of widely spaced cycles all for the benefit of melodic clarity. Similar too is the greater use of contrary movement such that the contour does not rise or fall too quickly. Moreover, the length of the metcycles is due to the composer's conscious choice of cyclic succession based on evaluative judgment of melodic resultants. Melodically constructed lines usually possess a greater percentage of stepwise motions, as is the case here. Specifically, the composer has chosen successions resulting in voice movement by 0, 2, or 4. This imparts a dream-like ambiance to this section. Essentially, each horizontal line traverses a whole-tone scale. Adjacent voices separated by the intervals of 3, 5, or 7, traverse different whole-tone scales, while voices separated by 2, 6, or 10 traverse the same whole-tone scale. In order to avoid monotony, an exception to this voice movement is made in measure 44, (from beat one to beat two) where movement by semitone is used. Effectively, this semitone movement causes some voices to switch allegiances from one whole-tone scale to another, which in-turn infuses variety amidst the uniformity. The restatement of the four-measure metacycle in measure 46 is cut short after only two measures.

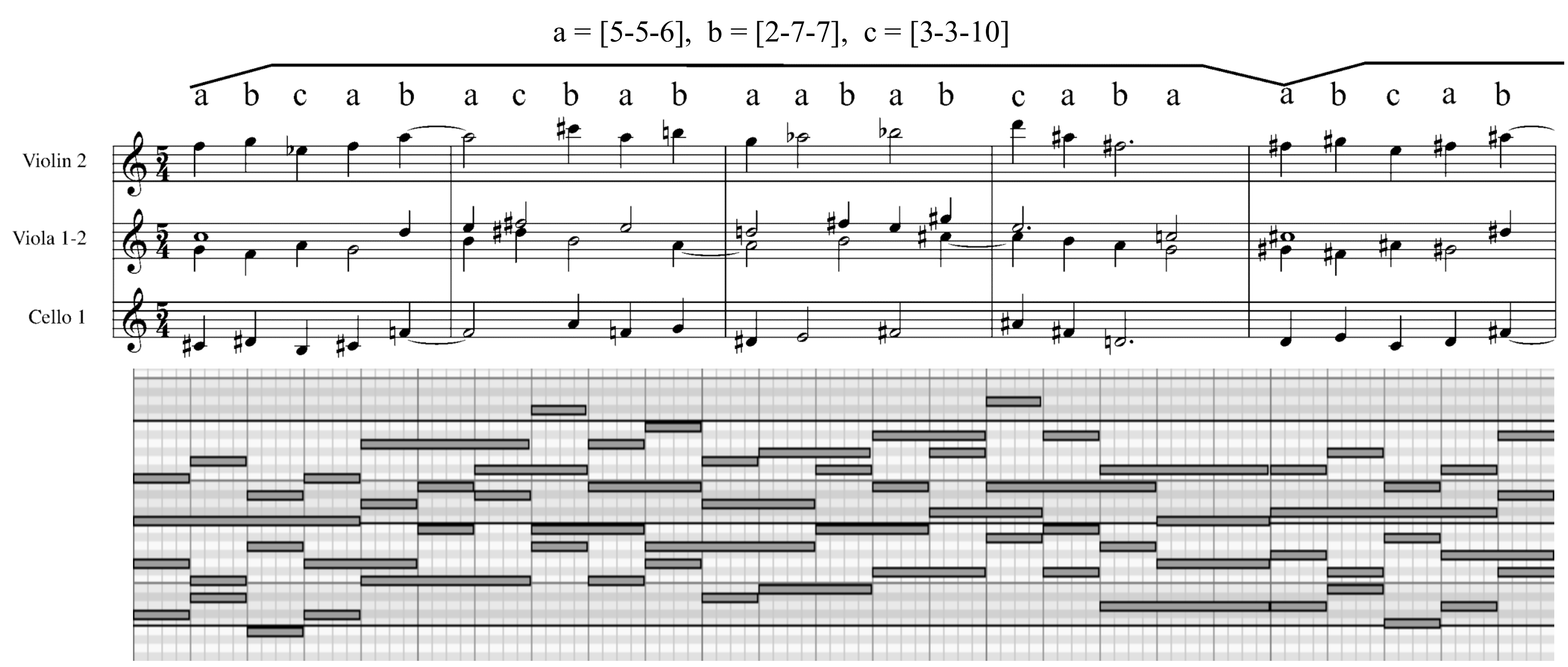


Figure 5.22. Excerpt from Movement II (mm. 42-46) consisting of metacycles from cc 16-3.

In measures 48-50, the contour descends as the texture is thinned to cycles from cyclic class 16-2. In contrast to the previous measures, horizontal movement consists of large melodic leaps (Figure 5.23). Specifically, a five-beat non-optimal metacycle is stated twice and consists of horizontal voices partitioning 12 by way of [5-7] cycles. Interestingly, the voice motion for the upper voice of the metacycle is (0, 0, -7, -5), while the simultaneous lower voice motion is (-5, -7, 0, 0), which is exactly retrograde of the upper voice. Also note that the pitch content of the metacycle (excluding the first two beats) conforms to the key of E major, with a nice cadence on [7-9] sounding like IV^7.

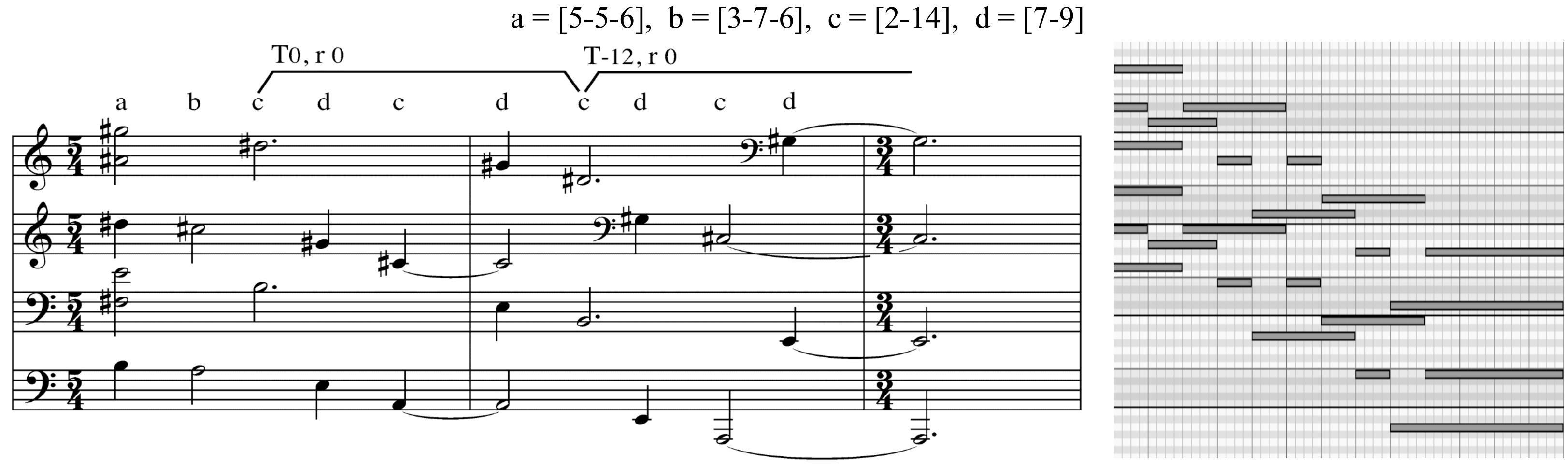


Figure 5.23. Excerpt from Movement II (mm. 48-50) showing simultaneous prime and retrograde voice motion.

The A^7 of measure 50 moves to a chromatic mediant C# major in measure 51, which is actually a single stacking of a pedal [7-9] cycle to start the concluding section F (mm. 51-59). Above this pedal chord is a pattern of cycles from resulting from wedge voice leading. These cycles are members of cyclic classes 16-2 and 16-4, which are able to interact due to a suspension of the voice crossing rule. Specifically, cycles of larger cardinality move inwards and converge onto voices of smaller cardinality and vice versa. Voice crossing is allowed, since two voices may merge inward onto a unison, or two unison voices may branch outward.

This wedge voice leading technique can be seen in Figure 5.24. The basic pattern is an alternation between cycles c-d-a (prime) and a-d-c-e (retrograde with extension), all in non-rotated form. Within individual patterns, voice movement is strictly by semitone, whereas voice movement between individual patterns is by leap in parallel or similar motion downward. The transpositional outline of the entire section is: T0, T-4R, T-7, T-10R, and T-13 (where R indicates the retrograded and extended version). The distance between each consecutive pair of patterns is as follows: -4, -3, -3, -3.

Thus, we can distinguish (or group together) the first pair of patterns on the one hand, from the last three patterns (essentially an augmentation) on the other hand. Only the last three patterns are shown in Figure 5.24. Three pick-up notes precede each group, as seen in measure 53 below. After a cadence on cycle [7-9] in measure 56, a scalar treatment of cycle [2-3-2-3-3-3] (cc 16-6) descends onto a final cadence on another [7-9] cycle (mm. 58-59), drawing Movement II to a close.

a = [7-9], b = [5-11], c = [4-3-4-5], d = [2-5-2-7], e = [1-6-3-6]

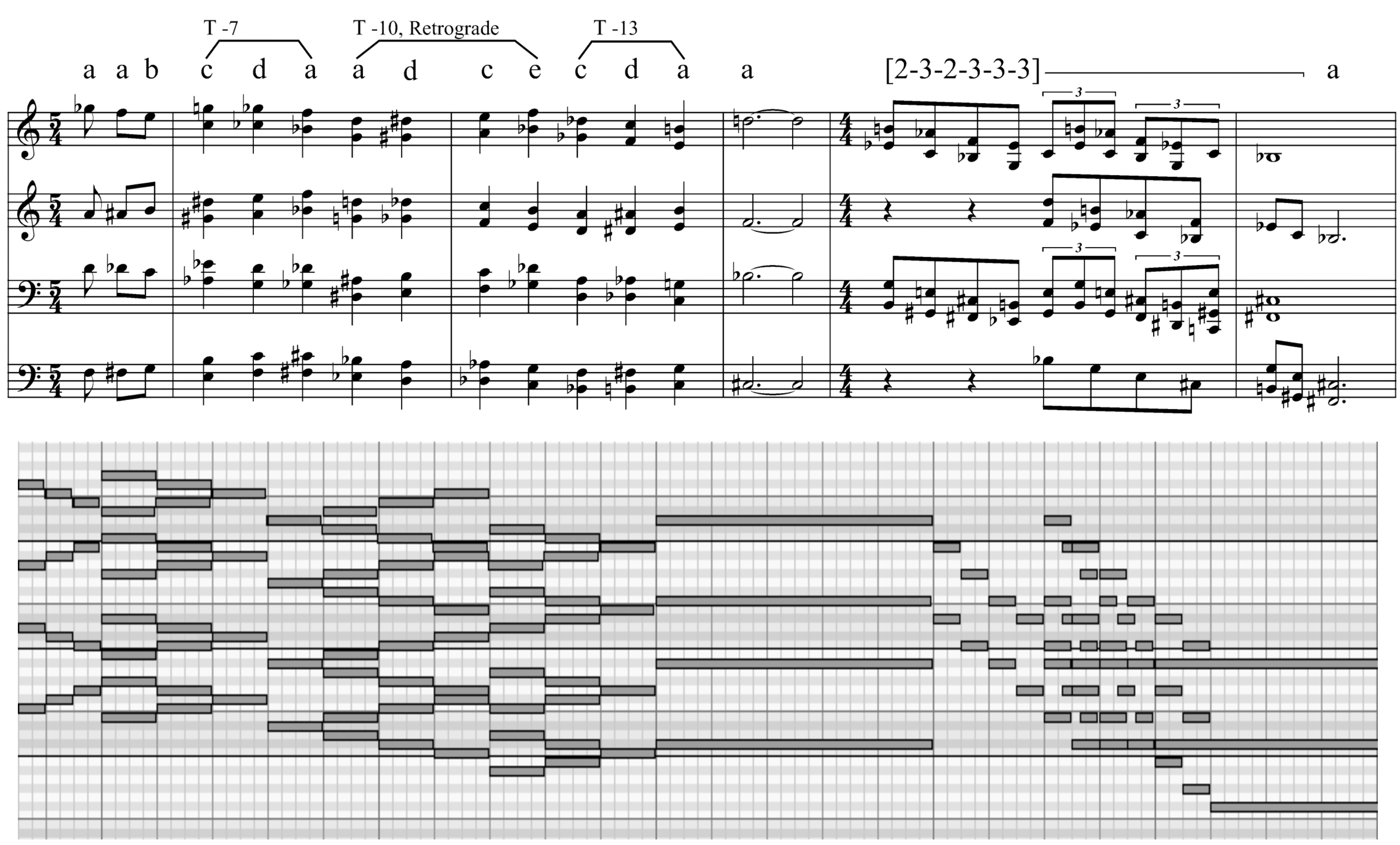


Figure 5.24. Excerpt from Movement II (mm. 53-58) illustrating wedge voice leading technique.

<u>Movement III: *Allegro*</u>

Movement III, *Allegro* is constructed from cycles from prime-number classes 3, 5, 13, and 17. The tempo of ♩. = 76 (♩ = 115) is maintained throughout, but may certainly be faster if the conductor so desires. The through-composed form is A^1, B^1, C^1, D^1, B^2, E, F, A^2, C^2, D^2 (Table 5.3 a). From the background (prime-number class) perspective, the macroform is a three-part A B C wherein each part consists of microforms (prime-number classes) arranged into arch-like constructions (Table 5.3 b).

Table 5.3. Formal Layout of Movement III: *Allegro*.

a. Cyclic class perspective

Form:	**A^1**	**B^1**				**C^1**	
Prime-Number Class:	3	5	13	5		3	13
Cyclic Class:	18-4	15-4	13-4	15-4	20-3	24-4	13-3
Measure Numbers:	1-9	10-16	17-22	23-35	36	37-44	45-46

D^1	**B^2**	**E**	**F**		**A^2**
17	5	13	17	13	3
17-5	15-4	13-3	17-3	13-4 and 13-5	18-4
46-56	57-70	71-78	79-88	88-91	92-101

C^2				**D^2**		
3	13			17		
24-4	13-2	13-3	13-1	17-5		
102-108	109	110-120	121-122	123-129	130-138	139-146

b. Prime-number class perspective

Macro Form:	**A**					**B**				**C**				
Micro Form:	**a^1**	**b^1**	**c^1**	**b^2**	**a^2**	**C^2**	**d^1**	**b^3**	**c^3**	**d^2**	**c^4**	**a^3**	**c^5**	**d^3**
Prime-Number Class:	3	5	13	5	3	13	17	5	13	17	13	3	13	17
Measure Numbers:	1-44					45-78				79-146				
Number of Measures:	44					34				68				

Mirroring the first movement, section A^1 (mm. 1-9) is orchestrated primarily for strings alone. It begins with octave F#s, (mm. 1-2) in the bass, which in measure 3 lead into a semi-functional bass line underneath metacycles from cyclic class 18-4. In fact, a strong bass line is featured at various points throughout this movement. In lengthened rhythmic values, this bass line adds stability and interest to the bustling activity of the wave-contoured metacycles. As Figure 5.25 shows, the pitch content of initial metacycle is continuously transposed by +7 and rotated +3 upon subsequent restatements. The metacycles are optimal since four rotations (equal to the cardinality) are required for return to the initial cycle in rotation 0. Two metacyclic completions are achieved, and just at the point where one would expect a third to commence (the la of beat three, m.4), cycle [3-5-3-7] is transposed up by semitone vis-à-vis all-parallel motion, effectively interrupting the pattern. Note that metacycles themselves avoid all-parallel motion. This transposition technique is used quite often throughout this movement and elsewhere in the symphony to break up and vary the predictability and pitch content of established metacycles. Thwarting expectation is a hallmark of good symphonic writing. The arrows in Figure 5.25 show instrumental entrances. The pitches before the arrows are not present in the final orchestration, since the texture here builds from one voice (m. 3) to five voices (m. 4). A wave-like contour alternates from ascending to descending (retrograde) throughout measures 3-9, with one more semitone shift in measure 6 (on beat three).

a = [3-5-3-7], b = [4-4-5-5]

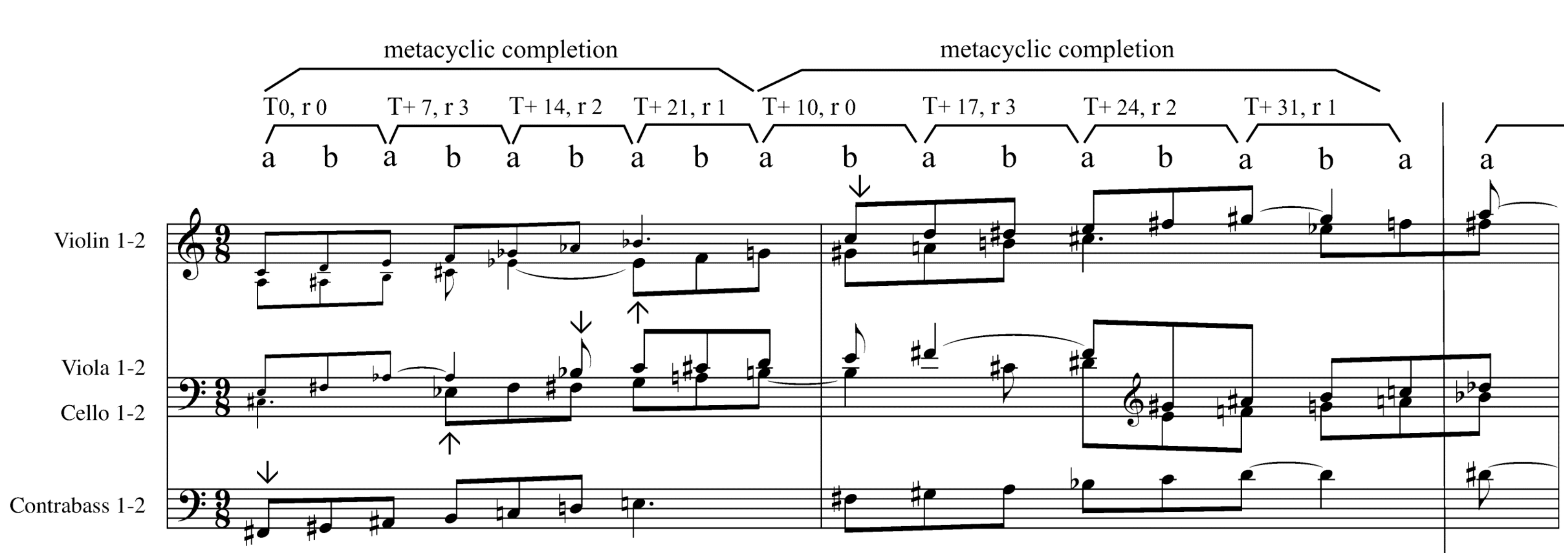


Figure 5.25. Excerpt from Movement III (mm. 3-4) comprised of cc 18-4 metacycles.

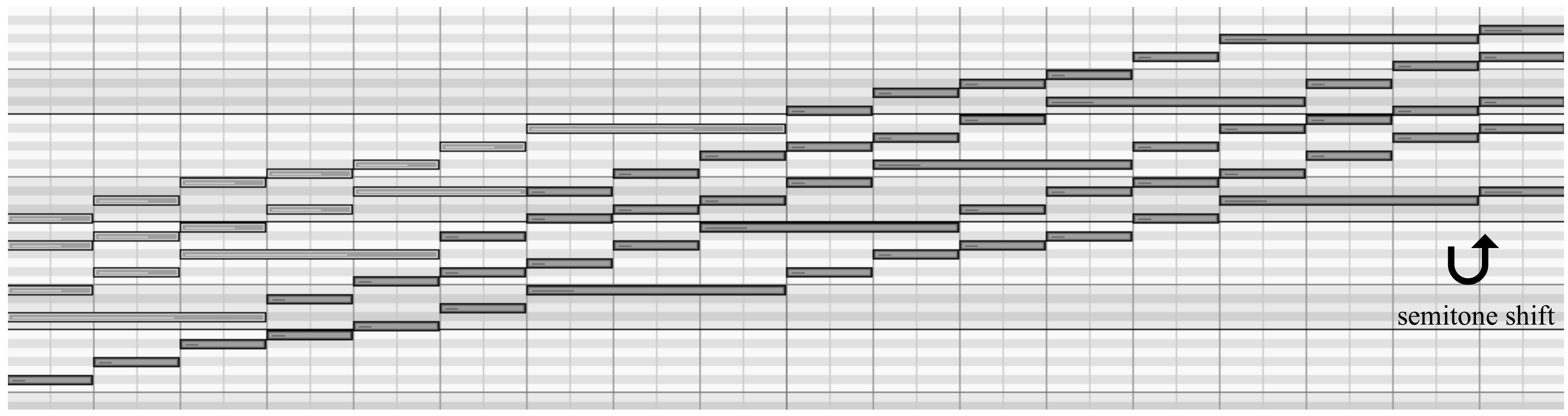


Figure 5.25. Continued.

Section B[1] (mm. 10-36) begins in measures 10-16 with metacycles from cc 15-4. As shown in Figure 5.26, canonic imitation occurs (entrances on low A#) since the voice movement per LCM equals the partition of the cycles (15). The metacycle is optimal, with every consecutive occurrence containing a rotation (+1), no transposition (0), and an added lower voice beginning on the reiterated A#. It is not until measure 12 that a cyclically adjacent pair becomes apparent: the A# and C# (15 semitones apart). The [2-5-3-5] cycle in measure 13 (beat three) is transposed +4 on the te of three to begin retrograded statements (mm. 13-16) of the same metacycle. Like Figure 5.25, this all-parallel motion interrupts the established pattern. The overall contour here is a sharp ascent followed by a similar descent, with a peak on the high F in measure 13.

a = [2-5-3-5], b = [2-4-2-7]

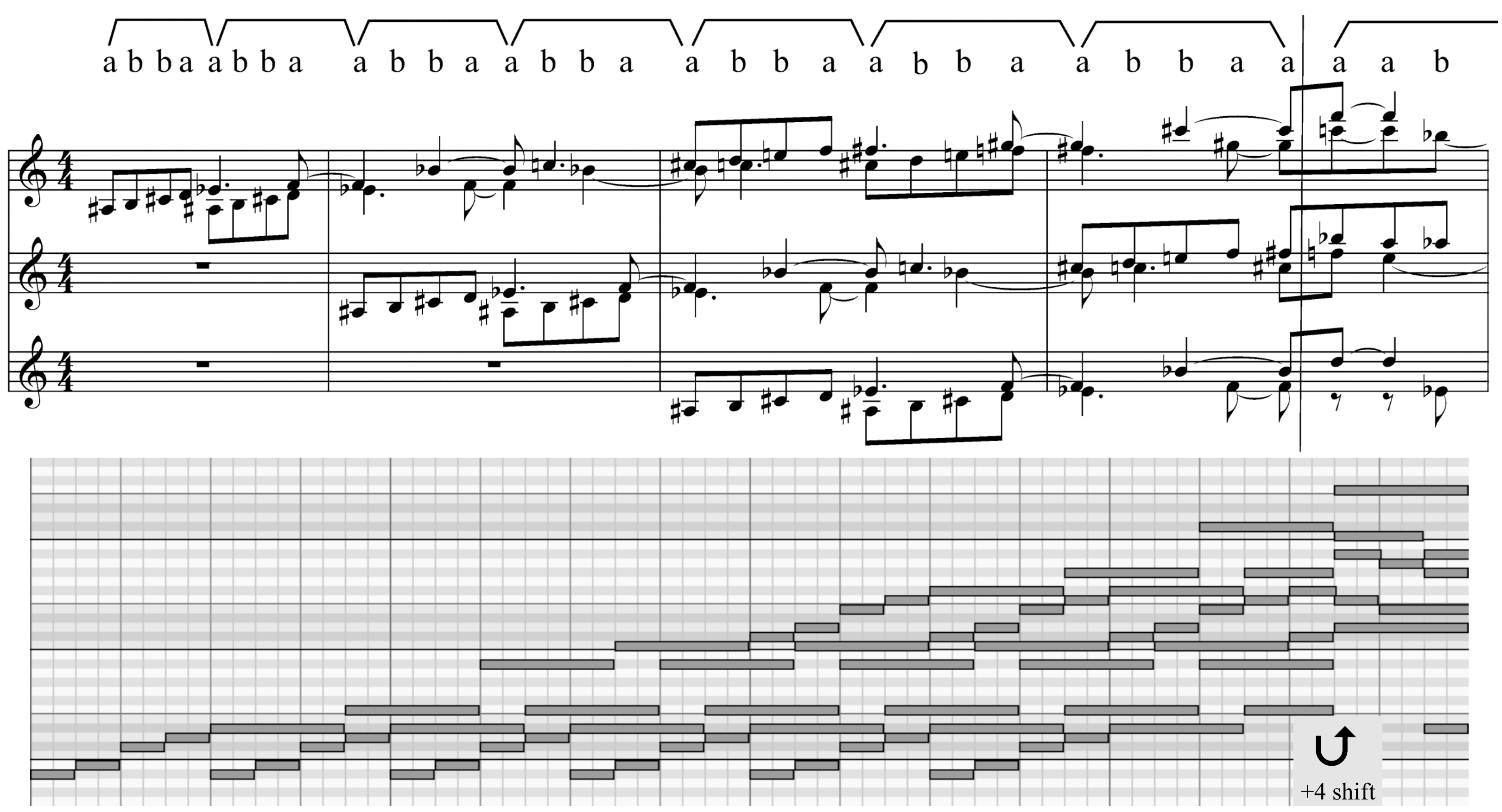


Figure 5.26. Excerpt from Movement III (mm. 10-13) showing canonical metacycles from cc 15-4.

A short bridge passage occurs in measures 17-22 consisting of chromatic lines traversing vertical cc 13-4 cycles. Section B[1] continues in measures 23-36 with an extended version of the same canonical metacycles seen in Figure 5.26. The texture here is thicker and contains more transpositional interruptions by all-parallel motion. Section B[1] concludes with 'hammer chords' in measure 36 on a single, reiterated [4-7-9] cycle (PNC 5, cc 20-3).

Skipping ahead, section B[2] (mm. 57-70) is made up of a one-measure metacycle of cycles from cc 15-4. This is the same cyclic class as section B[1] as well as the same two cycles, but the particular metacycle (voice leading distances and directions) is different. The texture builds from two to five voices by a gradual rate of one voice per two measures. Figure 5.27 shows measures 63-66 at the point where, after a gradual building, the texture has arrived at five voices, the outermost of which are cyclically adjacent.

Every measure is a retrograde of and a semitone higher (by all-parallel motion) than the previous one. Each measure contains one complete statement of the metacycle followed (or preceded) by an incomplete statement five semitones away. Since the metacycle is not optimal, every instance is non-rotated. There is a rhythmic (written-out) ritardando in measures 66-70, the beginning of which can be seen in 66 below. The horizontal motion per metacycle (per stacking) from top to bottom is [2-1-1-1], [0-0-3-2], [5-0-0-0], and [0-2-3-0], thus each partitions 5. Note that the motion of voices two and four are retrograde related (per metacycle) and their rhythms are also retrograde related (per measure). These rhythmic values are exchanged every measure.

a = [2-4-2-7], b = [2-5-3-5]

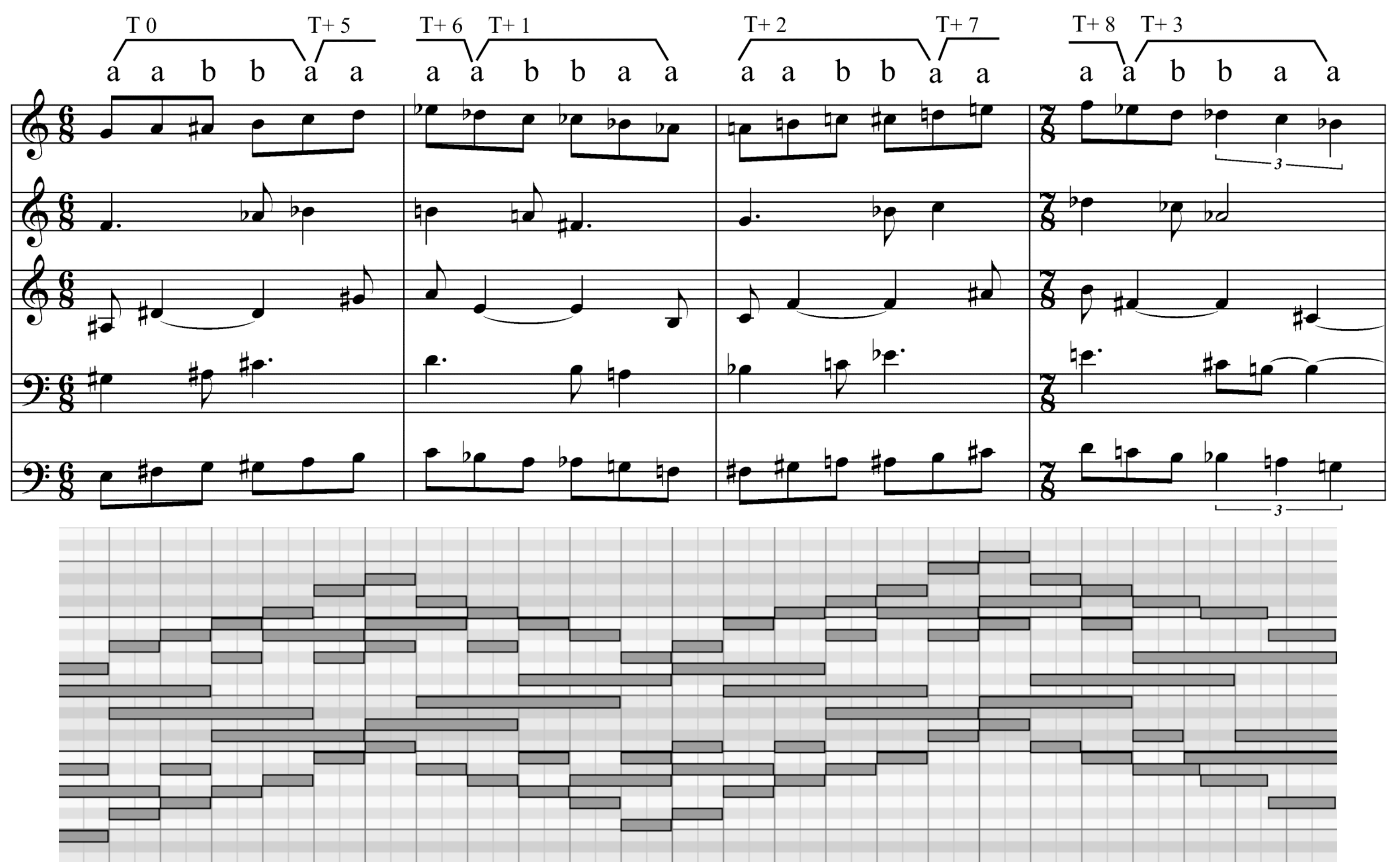


Figure 5.27. Excerpt from Movement III (mm. 63-66) illustrating non-optimal metacycles from cc 15-4.

In the climactic section F (mm. 79-91), a simple one-measure metacycle is repeated at different transposition levels every measure and using any one of three possible rotations (see Figure 5.28). The metacycle consists of two alternating cycles from the widely spaced cyclic class 17-3: [3-7-7] and [3-3-11]. The wide spacing of the cycles produces pleasant and uncluttered sonorities. It is orchestrated for six voices, which is one voice shy of two stackings of the cyclic pattern.

Adjacent metacycles are not connected via voice-leading rules, but rather by abrupt shifts of similar or parallel motion (they are not interlocked). The abruptness of these shifts is lessened somewhat by that fact that adjacent metacycles are in a different rotations. Since the cardinality of these cycles is 3, there are three possible rotations of the metacycle. The arrows at the bottom of Figure 5.28 show the measure-to-measure transpositional distance (see also Table 5.4 below).

Silence (in the form of an eighth-note rest) is inserted at the beginnings of various measures throughout this section (measures 79, 81, 84, and 86). This silence is either accompanied by a remetering to 10/8 or an eighth-note is subtracted from the beginning of the metacycle to make room for it. These variation techniques ensure variety amidst uniformity. The horizontal movement is dominated by whole steps in contrary and similar motion, but completely free of sustained motion. Thus, every voice contributes to a flowing and intensely energetic character appropriate for a fast tempo. A muscular, semi-functional bass line in long rhythmic values (not shown) adds stability to the fast-moving metacycles.

a = [3-7-7], b = [3-3-11]

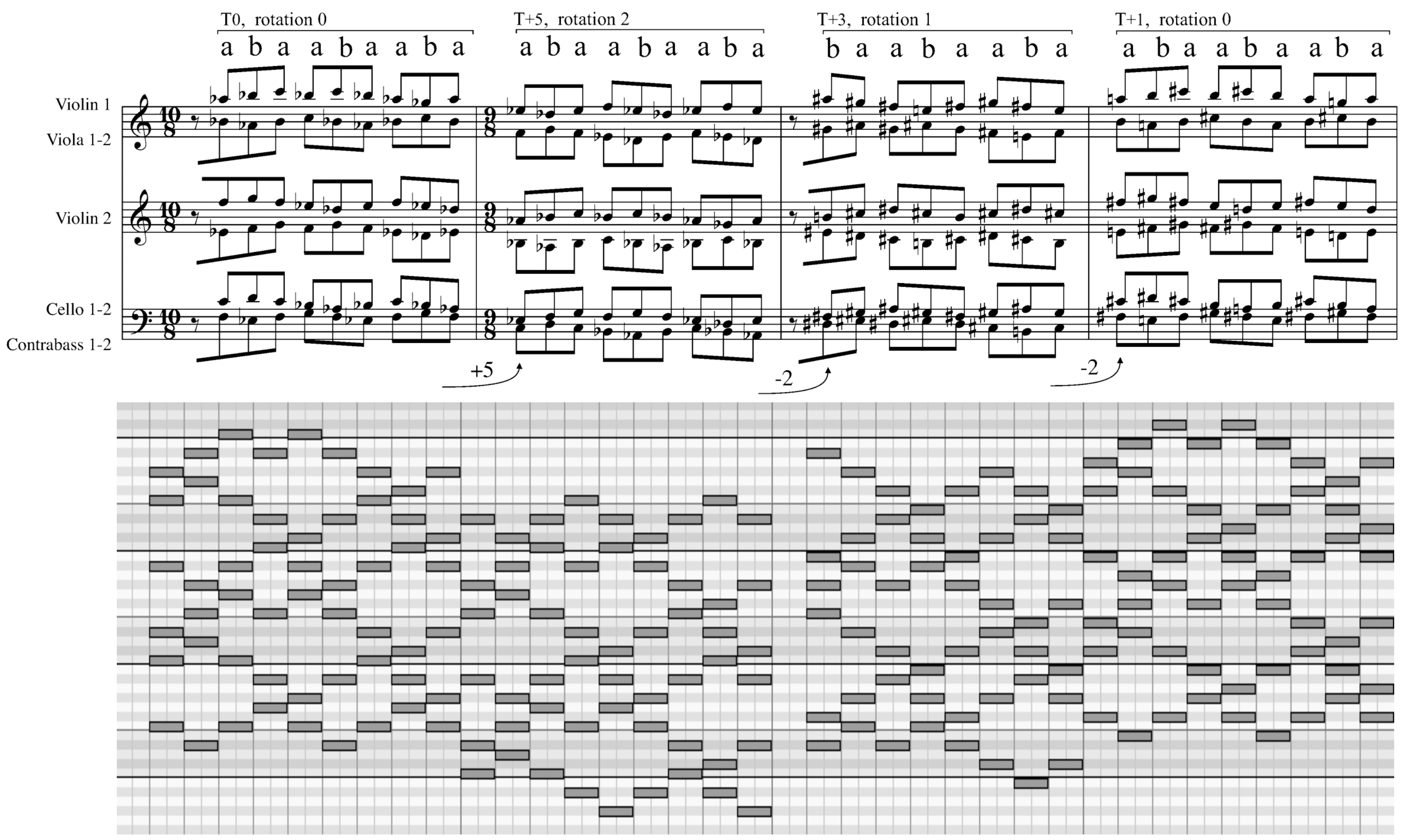


Figure 5.28. Excerpt from Movement III (mm. 79-82) consisting of metacycles from cc 17-3.

Table 5.4 shows transpositional relationships and rotations for occurrences of the metacycle in section F.

Table 5.4. Transpositional Relations and Rotations in Section F.

Transpositional relation to initial measure:	T0	T+5	T+3	T+1	T-1	T+8	T+16	T+13	T+3
Rotation:	0	2	1	0	2	0	2	2	1
Transpositional relation to previous measure:	0	+5	-2	-2	-2	+9	+8	-3	-10
Measure number:	79	80	81	82	83	84	85	86	87

Section D2 (mm. 123-146) is comprised almost exclusively of cyclic class 17-5. Figure 5.29 shows the dominant metacycle for this section. Although very short, it is optimal, which engenders five rotations for every one measure. Horizontal voice motion is either by semitone or by common tone and follows the one-measure pattern [1, 0, 1, 1, 0, 1, 0, -1, -1, 0], which results is a transpositional increase of only one whole-step per five metacycles. The chromatic nature of the horizontal lines provides source material well-suited for a compound melody and bass line (which are functional at their registral placements). These outer lines are formed from material derived from two voices (which have since been discarded). Thus the metacycle acts as a disposable referential construction from which instrumental lines may be created. Had the source material been more widely spaced, or had the original voices moved at a higher rate, the compound melody would have been characterized by a greater incidence of large and awkward leaps disadvantageous to good melodic writing. As it stands, the compound melody has a good balance of directionally opposed steps and leaps. The character of this final section it forward driving and bombastic, to bring *Symphony No. 1* to a rousing conclusion.

a = [2-2-4-5-4], b = [1-4-4-4-4]

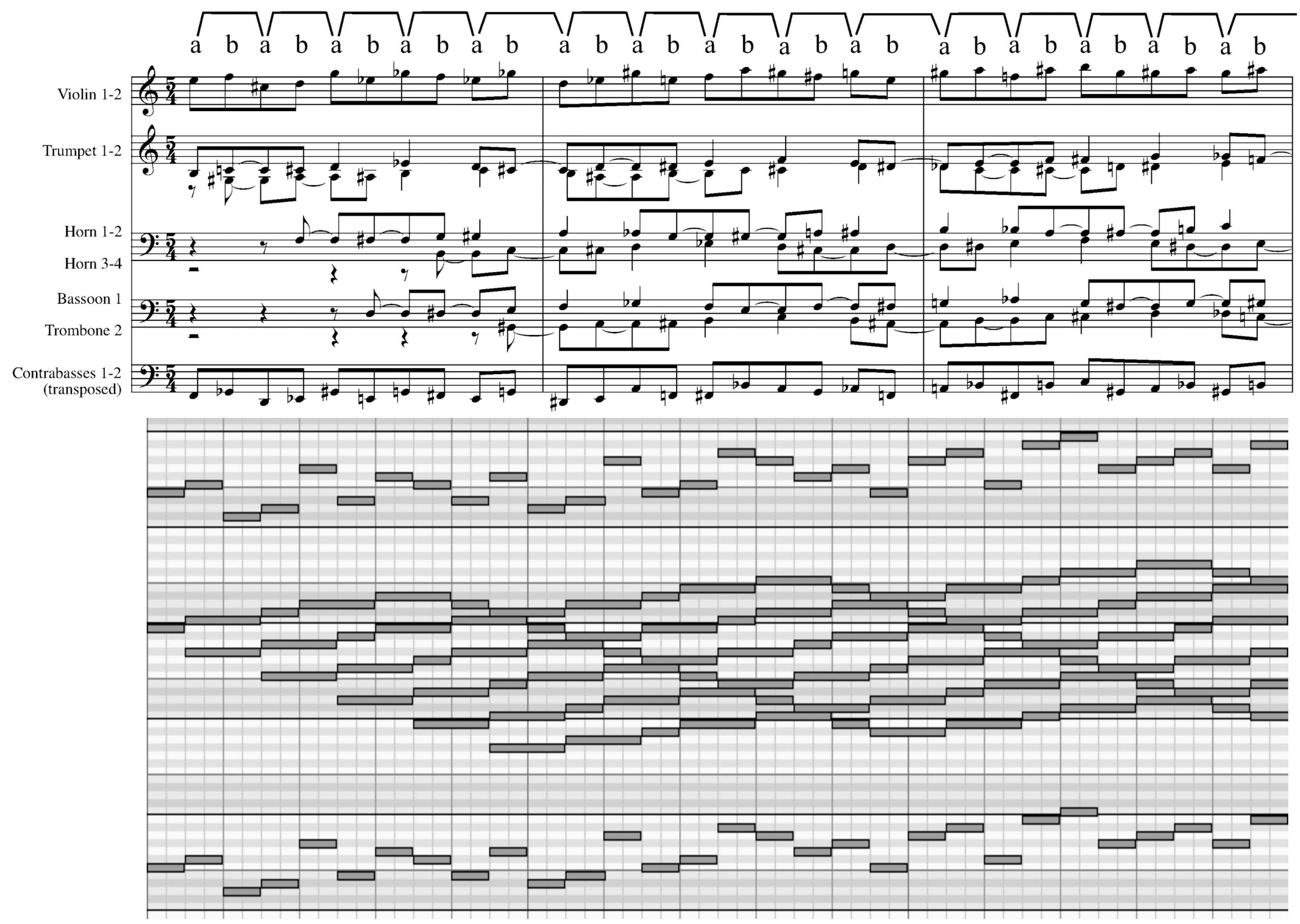


Figure 5.29. Excerpt from Movement III (mm. 123-125) showing compound outer lines.

CHAPTER VI
SUMMARY AND CONCLUSIONS

While the initial experiments with intervallic constructions privileged no particular constructions, it soon became apparent that certain arrangements of intervals – interval cycles – merited serious attention. It was deemed arbitrary to restrict interval cycles to partitions of 12, so other partitions were examined. Patterns and invariances were determined to behave predictably under certain conditions, a discovery that lead to a new set of equivalence classes uniquely suited to interval cycles and engendering multilevel coherence. Clearly, it scarcely matters whether or not this coherence is "heard," only that it binds together musical objects of incredible complexity. What has been unearthed here is nothing less the logical extension of tonality. The constructions used in tonality – partitions of 12 – were simply extended to include partitions of other numbers. However, not all cycles are considered to be equal in usefulness, so filtering techniques were considered to narrow the infinite number of cycles to a more practical level and to maximize variety. The voice-leading techniques of tonality were extended to accommodate these non-functional constructions. Successions of verticalizations were not limited to considerations of function, but rather to considerations of voice leading motion, distance, direction, contour, and melodic character. It is questionable whether functionality, when applied to atonal interval cycles, is even a meaningful concept. After all, the idea of telegraphing the next chord has little "appeal."[19] But certainly, classifications similar to functions could be (or have already been) applied to atonal sonorities. Ignoring "root motion" for the moment, one might consider more dissonant interval cycles (perhaps those containing larger numbers of interval class 1 and 6) as requiring a resolution, thus might take on a 'dominant' function, leading to more evenly-spaced 'tonic' cycles: [1-3-7] going to [3-4-4] for example[20]. There are obvious problems with this sort of adaptation. From the relativist position comes the first objection: what is dissonant for one individual may not be for another. From the pragmatist position comes the second objection: it is entirely possible that dissonance does not 'require' resolution at all. To the antiestablishment composer, such objections would only serve as endorsements, so are better left unspoken. Alas, it seems doubtful that antiquated notions such as the necessity of functionality will completely disappear. If history has taught us anything, it is that the traditionalist voice is always loudest. The compositional method espoused here shouldn't upset such a voice, since it validates tonality as a subgroup. Yet, the doors are wide open for further research and applications. An obvious application of interval cycle manipulation is microtonal music. This method would certainly be able to organize structures outside of the twelve-tone scale. The obvious adjustment would be for the composer to remain conscious of just what interval partitions the octave, if any. Along empirical musicological lines, a study should be undertaken to determine the perceptibility of prime-number classes, cyclic families, cyclic classes, and so forth. Furthermore, a computer program is without doubt a logical next step. Such a program would drastically speed up the process of filtering and selecting cycles and metacycles within a set of user-chosen parameters. It may prove to be compositionally significant to consider this method as methods have been generally considered by some in the past: namely, as something to be avoided. I am not being facetious. To be clear, there are several avenues toward coherence advocated here, but one can certainly create coherence by what is *not* done. One might choose to consistently *avoid* cycles that map onto each other in any way. A future composition could be constructed entirely of cycles that have nothing at all to do with one another. Doing this on a consistent basis throughout a composition would certainly create coherence in itself, and is worthy of being attempted. In order to be successful, one must know exactly how different cycles relate to one another, and this document is a tool for doing just that. Ironically, but not unexpectedly, this document would be indispensable for those wishing to avoid related cycles. A composer can precisely control all of the parameters elucidated in this document, allowing a complex web of relations to accrue. A full understanding of cycles like the one presented here can enhance and inform further creative and theoretical activity.

---

[19] What chord do you think is coming after the V^7?

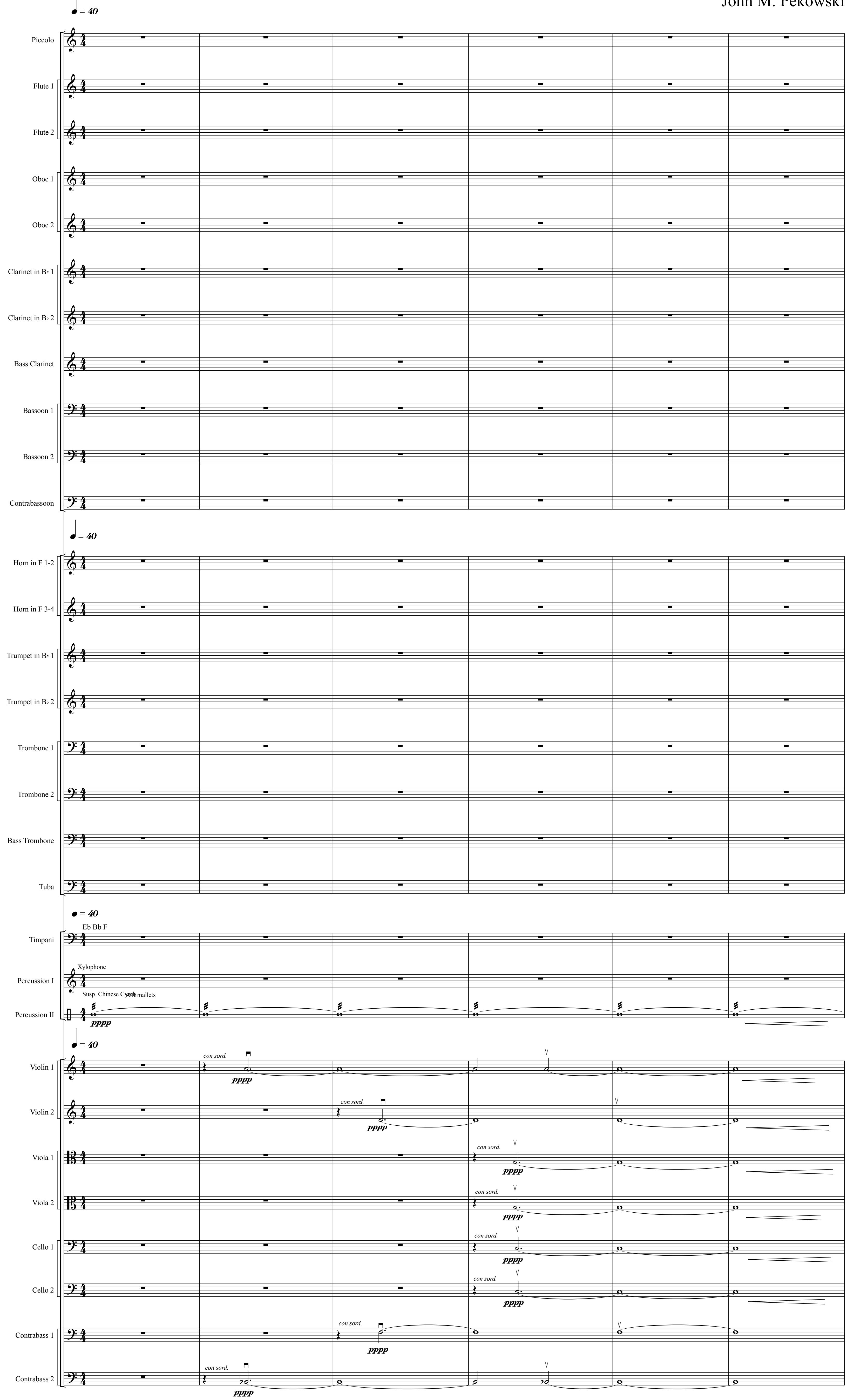
[20] Test this progression on a keyboard. Does it not sound like a dominant-tonic relationship?

CHAPTER VII
SCORE OF *SYMPHONY NO. 1*

Symphony No. 1

I. Adagio, Moderato, and Allegro

John M. Pekowski




7
Legato
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc. I
Perc. II
dampen w/cloth
l.v.
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2


A
13
= 46
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc. I
Perc. II
soft mallets
l.v.
Lg. tam-tam
with bow
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2
senza sord.
sul ponticello
sub.


25
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc. I
Perc. II
Lg. Susp. Cymb.
Vln. 1
E string
Vln. 2
A string
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2
Ord.
pizz.
gliss.


35
B
= 64
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
a 2
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
remove cloth
Timp.
Perc. I
Sm. tam-tam
l.v.
Lg. tam-tam
stop sound
Perc. II
Ord.
Vln. 1
Vln. 2
arco
A string
G String
Vla. 1
Vla. 2
Vc. 1
Vc. 2
D string
Cb. 1
Cb. 2


44
(♩= 97)
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
dampen w/cloth
remove cloth
Concert Toms
Perc. I
Metal Wind Chimes
Perc. II
A string
G String
Ord.
gliss.
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2


52
C
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc. I
Perc. II
Lg. Triangle
cup mute
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2


60
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc. I
Perc. II
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2
senza sord.
stopped
flutter
dampen w/cloth
remove cloth
pizz.


67
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc. I
Perc. II
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2
sub. pp
straight mute
Xylophone
rubber mallets
Susp. Chinese Cymb.
with bow
with mallet
l.v.
arco
pizz.
A string
gliss.


D
= 105
74
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc. I
Perc. II
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2
div. stopped
stopped
straight mute
gliss.
sim.
Brake Drums
D string
G String
div.
sub.
E string
A string


79
Marcato Pesante
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc. I
Perc. II
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2
Ordinario
a 2
senza sord.
sim.
Lg. Susp. Cymb.
wire brushes


Legato
87
E
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc. I
Perc. II
dampen w/cloth
l.v.
Susp. Chinese Cymb.
scrape with triangle beater from center to rim
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2
pizz.


94
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc. I
Perc. II
Concert Toms
dampen w/cloth
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2
arco
pizz.


103
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
stopped
Hn. 3-4
Ordinario
a 2
B♭ Tpt. 1
B♭ Tpt. 2
cup mute
senza sord.
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
dampen w/cloth
remove cloth
Perc. I
Perc. II
Sm. tam-tam
Lg. tam-tam
l.v.
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2
con sord.
gliss.


116
F
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Ordinario
a 2
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
senza sord.
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
gliss.
Timp.
Perc. I
Perc. II
Concert Toms
Metal Wind Chimes
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2

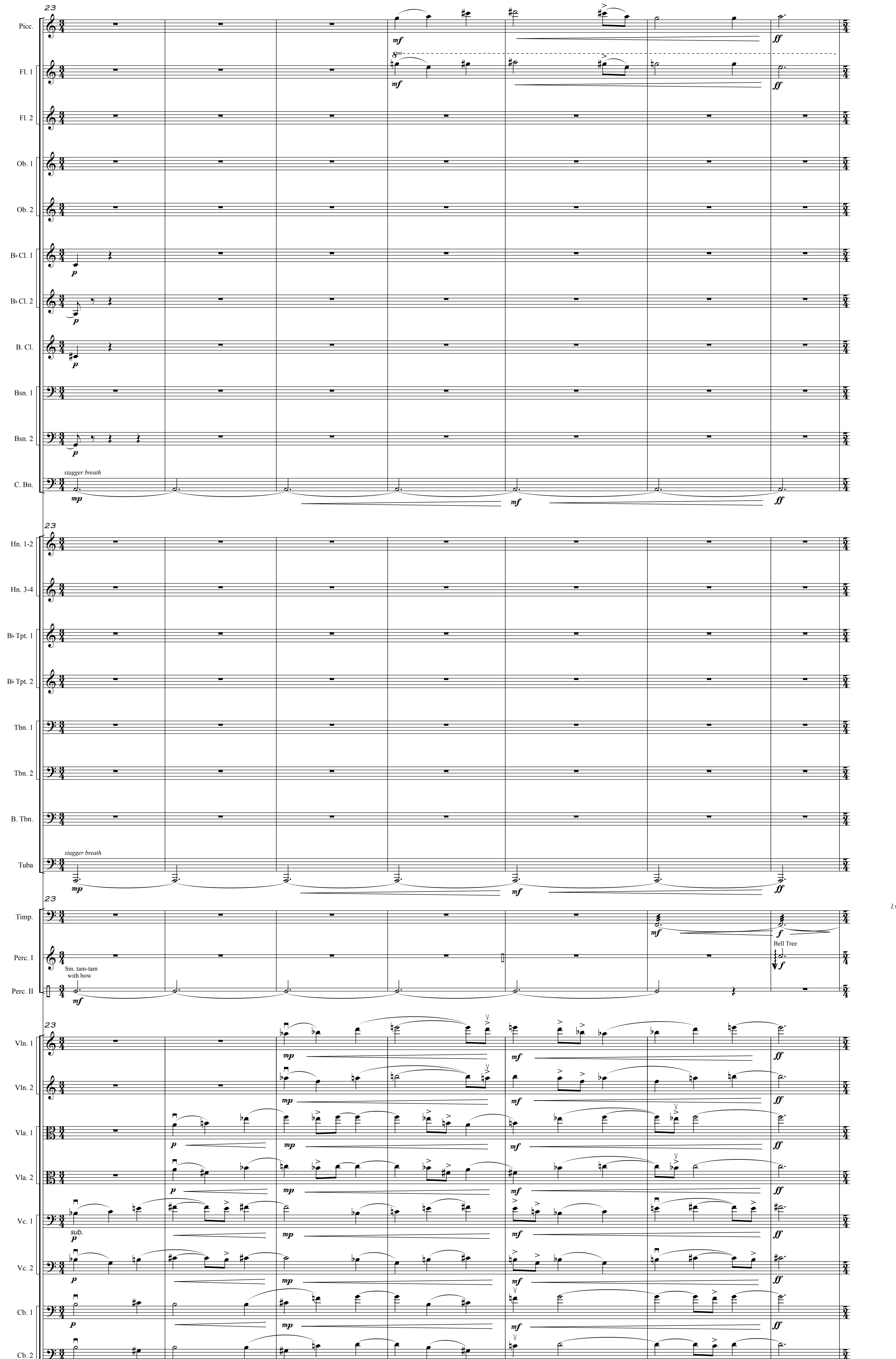

126
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc. I
Perc. II
l.g. tam-tam
with bow
as written
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2

Symphony No. 1

II. Lento


Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc. I
Perc. II
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2
A
bsn. 2
play
sub.
cup mute
senza sord.
damp
l.v.
Rain Stick
Lg. Susp. Cymb.
arco
pizz.


17
B
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
play
bsn. 2
sub.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
senza sord.
Timp.
Perc. I
Perc. II
Rain Stick
Concert Toms
Susp. Chinese Cymb.
l.v.
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2


23
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
stagger breath
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc. I
Perc. II
Sm. tam-tam with bow
Bell Tree
l.v.
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2
sub.


C
30
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
a 2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
tpt. 1
play
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc. I
Break Drums
Perc. II
Susp. Chinese Cymb.
with bow
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2


35
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
flute 2
play
bsn. 2
sub.
a 2
a 1
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc. I
Perc. II
l.v.
damp
Rain Stick
Sm. tam-tam
Lg. Susp. Cymb.
scrape with coin
gliss.
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2


42
D
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
F#-A-E
Concert Toms
Perc. I
Lg. Susp. Cymb.
Perc. II
l.v.
8va
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2
sub.


51
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
tpt. 1
tpt. 2
play
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
a 2
stopped
cup mute
Timp.
Perc. I
Perc. II
damp
Bamboo Wind Chimes
Lg. tam-tam
Bell Tree
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2

Symphony No. 1

III. Allegro


A
= 115
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
Hn. 1+2
play
B♭ Tpt. 1
B♭ Tpt. 2
tpt. 1
play
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
A# to B
Perc.
Perc. II
Lg. tam-tam
scrape with triangle beater
damp
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2


15
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc.
Perc. II
Bell Tree
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2
pizz.
arco


B
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
Hn. 1-2
play
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
gliss.
Tuba
Timp.
D# to E
Perc.
Perc. II
Thunder Sheet
Vln. 1
Vln. 2
Vla. 1
arco
Vla. 2
Vc. 1
Sul G
Vc. 2
Cb. 1
Sul E
Cb. 2


30
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
tpt. 1
play
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc.
Perc. II
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2
gliss.
sul ponticello
Sul E
ord.
play lower note if possible


37
C
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
slap tongued
sub.
Hn. 1-2
Hn. 3-4
stopped
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Triangle
Perc.
Perc. II
Vln. 1
Vln. 2
Vla. 1
Vla. 2
gliss.
Vc. 1
Vc. 2
ord.
pizz.
Cb. 1
Cb. 2


44
D With Energy
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
ordinario
Hn. 1-2
Hn. 3-4
open
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc.
Perc. II
Susp. Chinese Cymbal
damp
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2
arco
play lower note if possible


49
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc.
Perc. II
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2
gliss.
Brake Drum
pizz.
damp


56
E
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
clar. 2
play
bsn. 1
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
tpt. 2
cup mute
straight mute
gliss.
Timp.
Perc.
Perc. II
dampen w/cloth
Rain Stick
slightly invert and trickle
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2
sul ponticello
sul tasto
tutti
Freely
Solo 1st
rit.


64
F
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
play
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
gliss.
Tbn. 2
senza sord.
Tbn. 2
B. Tbn.
Tuba
Timp.
remove cloth
Perc.
Concert Toms
Perc. II
Metal Wind Chimes
Solo 1st
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
ordinario
Sul G
Vc. 2
ricochet
Cb. 1
pizz.
Cb. 2


72
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
play
Tbn. 2
B. Tbn.
Tuba
Timp.
F# to F
Perc. I
Perc. II
Crash Cymbals
roll
Vln. 1
all
Sul E
gliss.
Vln. 2
divisi
Sul D
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
sul ponticello
Cb. 2


G
79
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc.
Perc. II
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2
sfz
ppp
senza sord.
F to F#
Bass Drum
damp
crash


85
rit.
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc.
Perc. II
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2
B to A#
E to A
damp
Triangle
Lg. tam-tam
a 2
behind the bridge


92 H Fast, flowing, and connected
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc.
damp
Perc. II
Thunder Sheet
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
pizz.
Cb. 1
Cb. 2


99
I
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
oboe 1
play
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
bsn. 1
play
C. Bn.
Hn. 1-2
Hn. 3-4
Hn. 1-2
play
B♭ Tpt. 1
straight mute
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
B to A#
Perc.
Slap Stick
Perc. II
Lg. Susp. Cymb.
with bow
Vln. 1
gliss.
Vln. 2
Vla. 1
Vla. 2
Vc. 1
crisp
Vc. 2
arco
crisp
Cb. 1
Sul A
gliss.
crisp
Cb. 2
Sul E
gliss.
crisp


106
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
clar. 1
play
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
slap tongued
C. Bn.
Hn. 1-2
Hn. 3-4
senza sord.
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
F# to F
Timp.
Slap Stick
Perc.
Perc. II
gliss.
Vln. 1
Vln. 2
Sul G
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2


115
J
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
ord.
sub. mp
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc.
Perc. II
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2
pizz.
arco
Sul E
gliss.
gliss. 1st only
everyone else sustain


123
K
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
F to F#
Bass Drum
Perc.
Susp. Chinese Cymb.
with bow
Perc. II
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2


128
L
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc.
Perc. II
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2
F# to F
Metal Wind Chimes
sul ponticello
sul tasto
ord.
sub. p
sub. mp


134
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
A# to B
Perc.
Bass Drum
Perc. II
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2
ord.


138
M
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
gliss.
Timp.
Perc.
Perc. II
damp
Crash Cymbals
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2


142
Picc.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
B♭ Cl. 1
B♭ Cl. 2
B. Cl.
Bsn. 1
Bsn. 2
C. Bn.
Hn. 1-2
Hn. 3-4
B♭ Tpt. 1
B♭ Tpt. 2
Tbn. 1
Tbn. 2
B. Tbn.
Tuba
Timp.
Perc.
Perc. II
Concert Toms
Crash Cymbals
hands high
fff
damp
Vln. 1
Vln. 2
Vla. 1
Vla. 2
Vc. 1
Vc. 2
Cb. 1
Cb. 2

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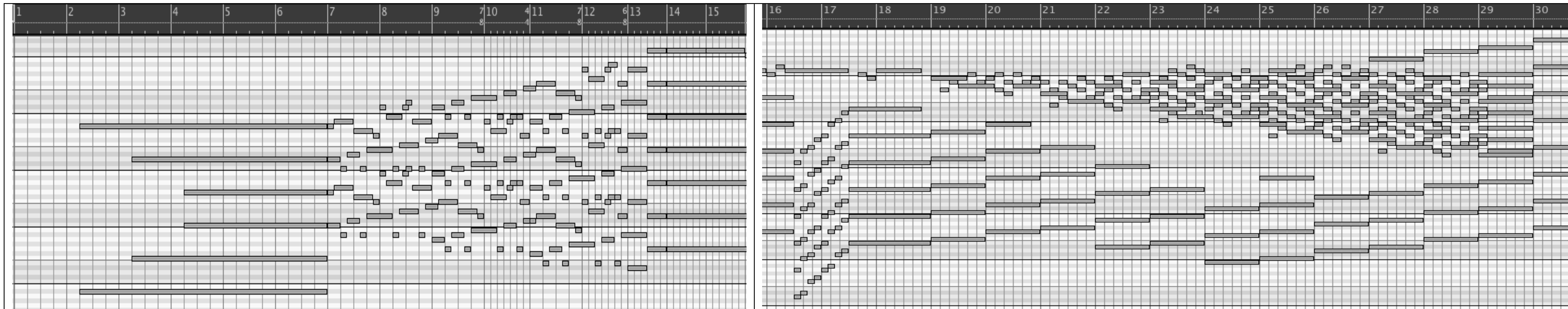
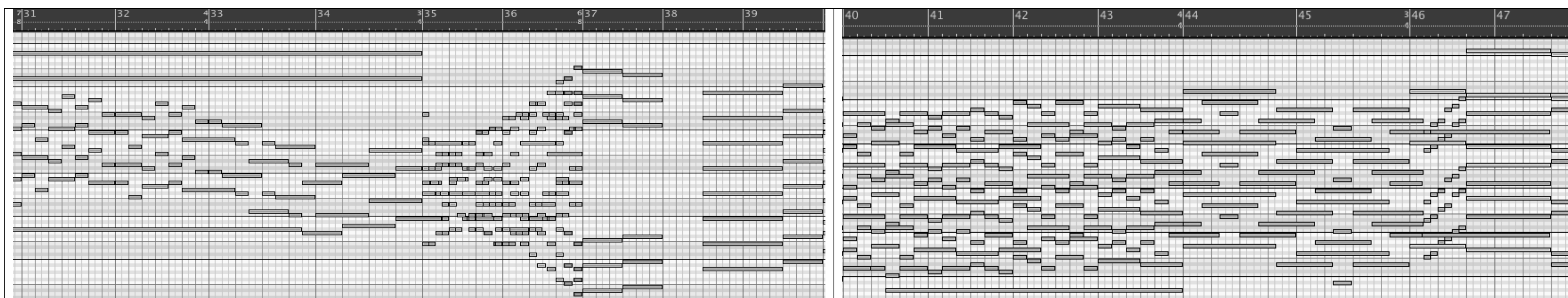
APPENDIX A
GRAPHICAL AND FORMAL CHART OF *SYMPHONY NO. 1*

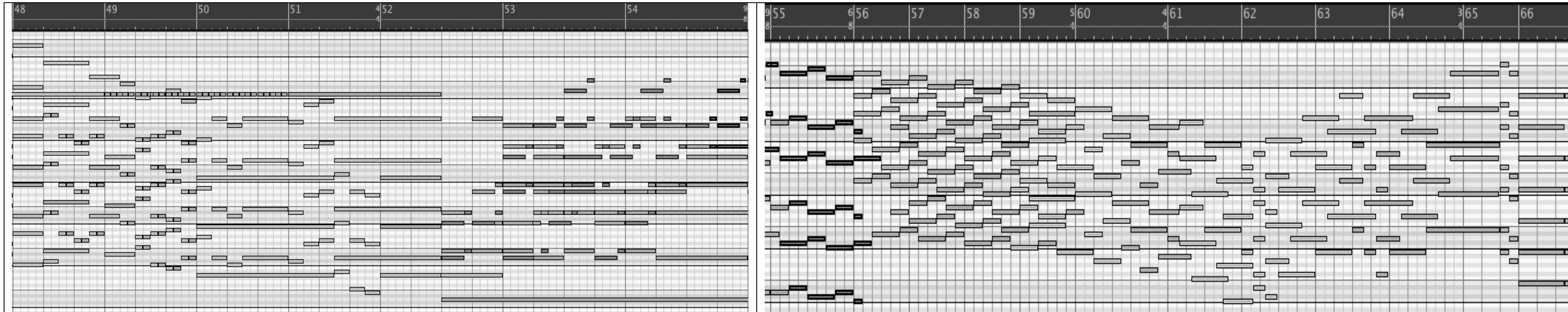


Form:	**A[1]**			**B[1]**				
Prime-Number:	7							
Cyclic Class:	7-1	14-2	7-1	7-4	7-1	7-3	14-5	7-1
Measure:	1-6	7-12	13-15	16-17	17-18	19-28	29	30

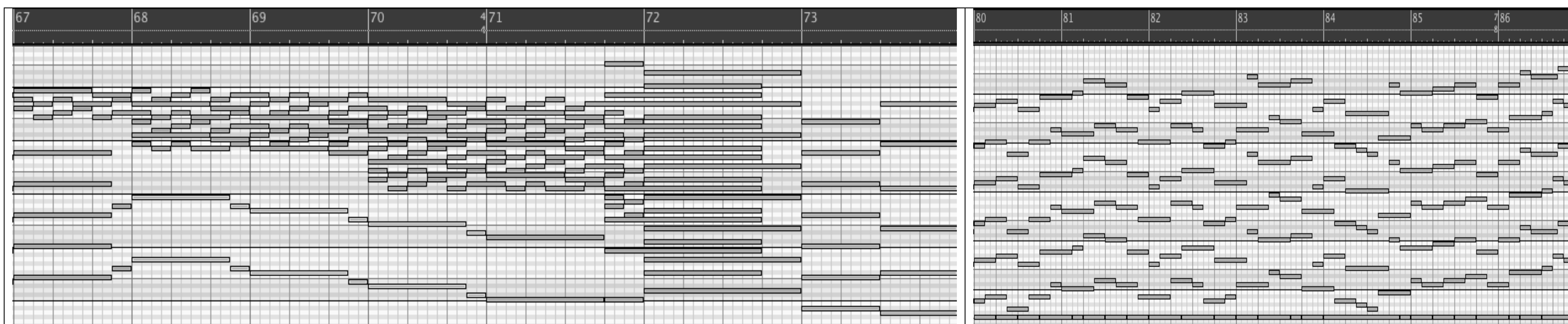


A[2]		**Bridge**	**C**		
7					
14-2	14-6	7-1	14-3	7-4	14-3, 14-2
31-34	35-36	37-39	40-45	46	47

Figure A.1. Graphical and formal chart of Movement I.

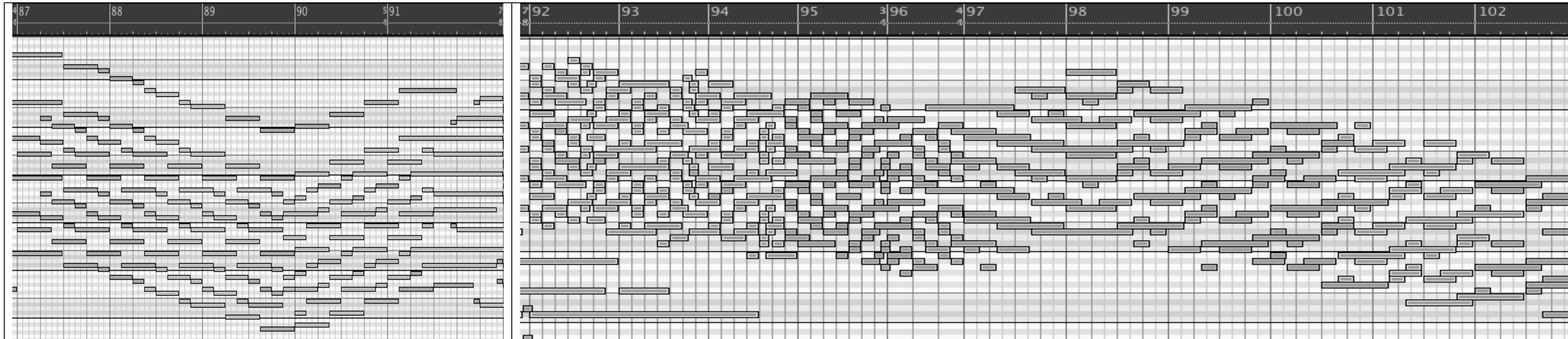


Form:	**A^3**		**Bridge**		**D^1**				
Prime-Number:	7		19		19			7	
Cyclic Class:	14-2	14-2, 19-4	19-4	14-2	19-2	19-4, 19-5	19-3	14-2	7-1
Measure:	48-50	51	52	53-54	55	56-59	60-65	65	66

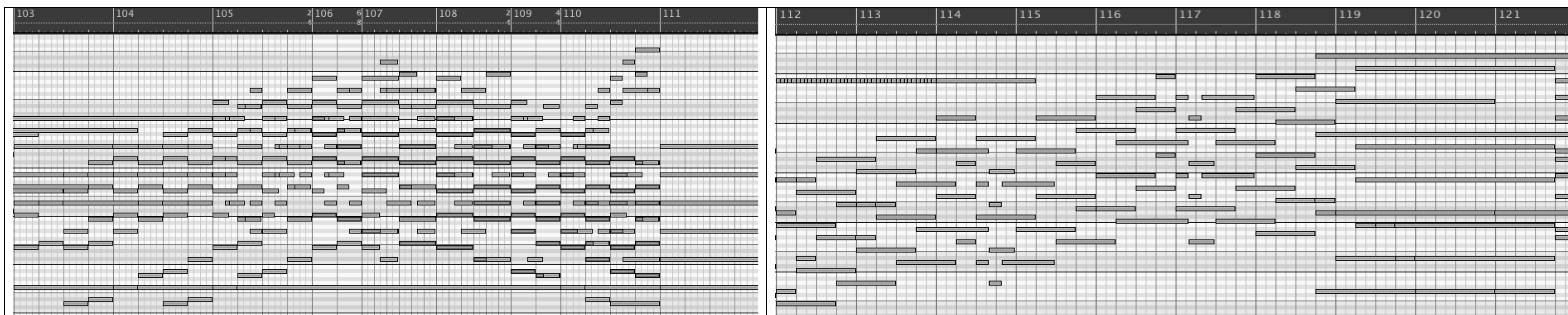


B^2			**D^2**
7		19	
7-3	7-1	19-2	
67-72	73		80-86

Figure A.1. Continued.

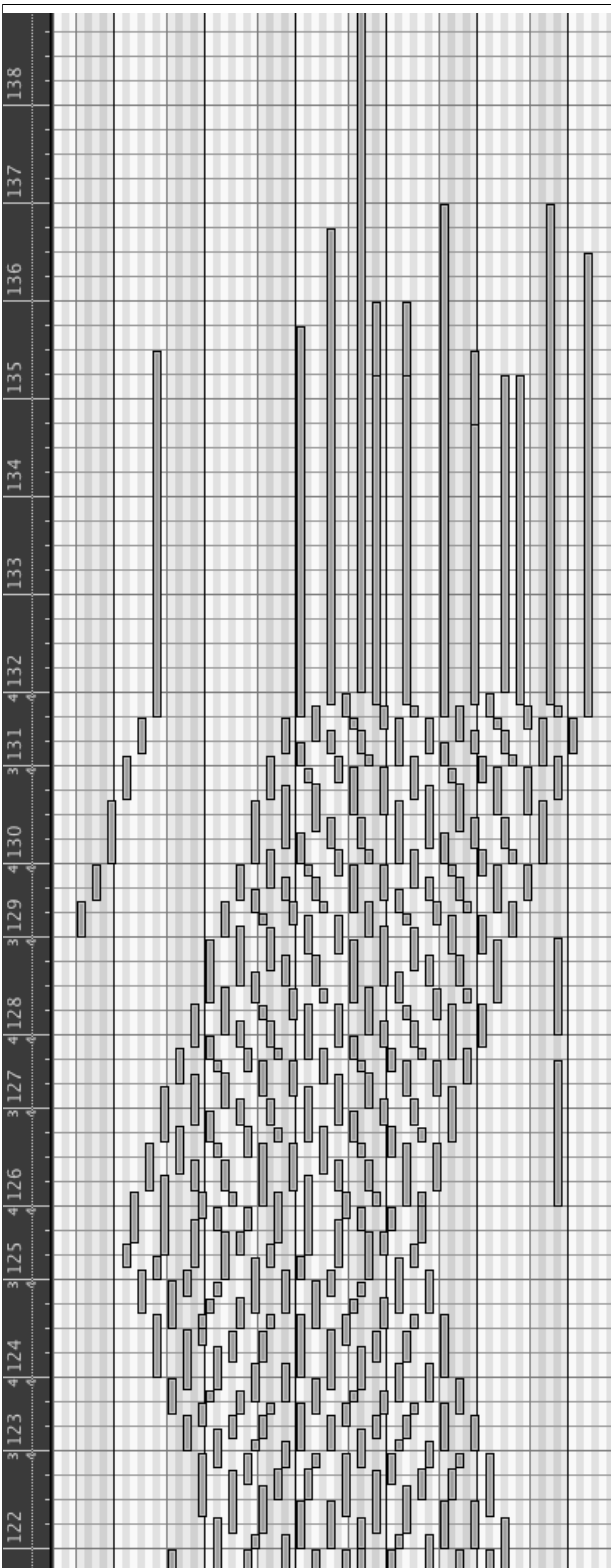


Form:	D^2 (continued)			E		
Prime-Number:	19		19, 7	7		
Cyclic Class:	19-3, 19-4	19-4	19-4, 14-3	14-6	14-5	14-4
Measure:	87	88-90	91	92-96	97-101	102



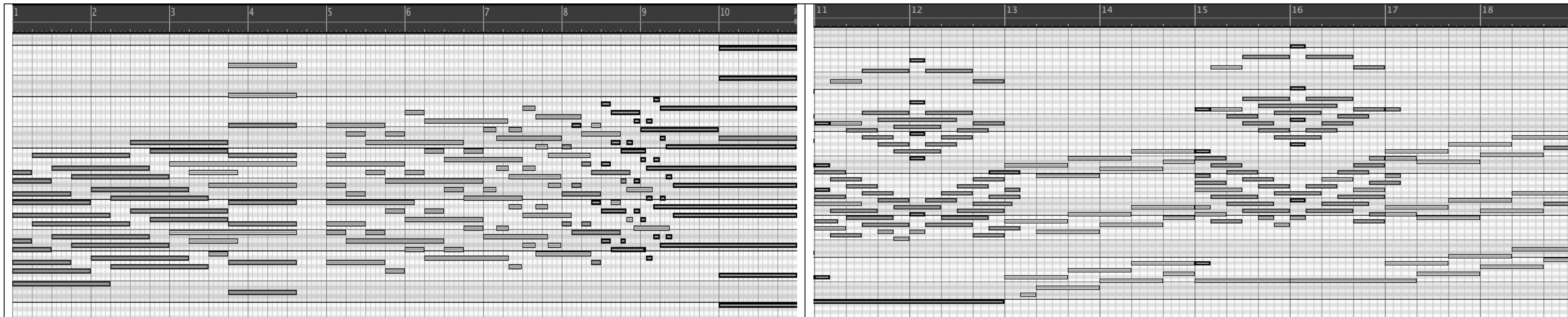
Bridge	D^3
7	19
7-3	19-3
103-111	112-121

Figure A.1. Continued.

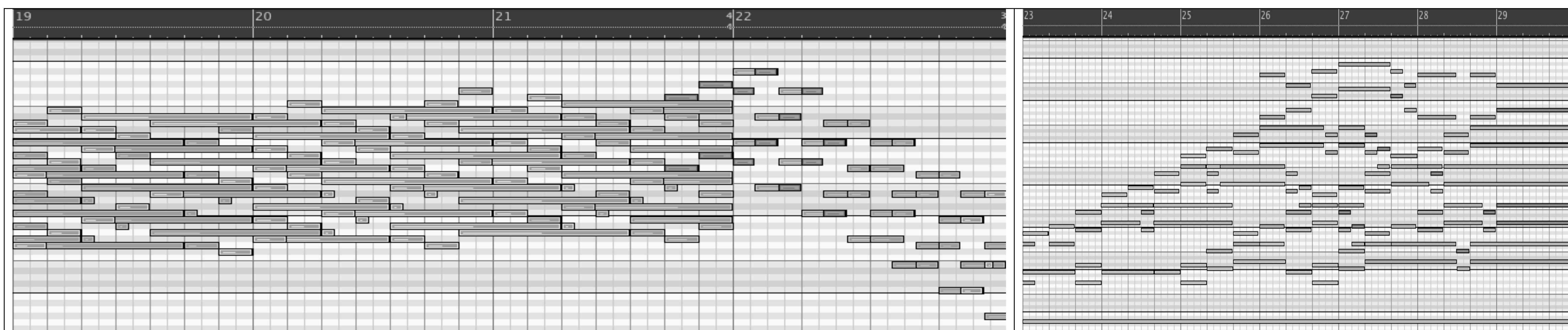


Form:	**F**
Prime-Number:	19
Cyclic Class:	19-5
Measure:	122-138

Figure A.1. Continued.

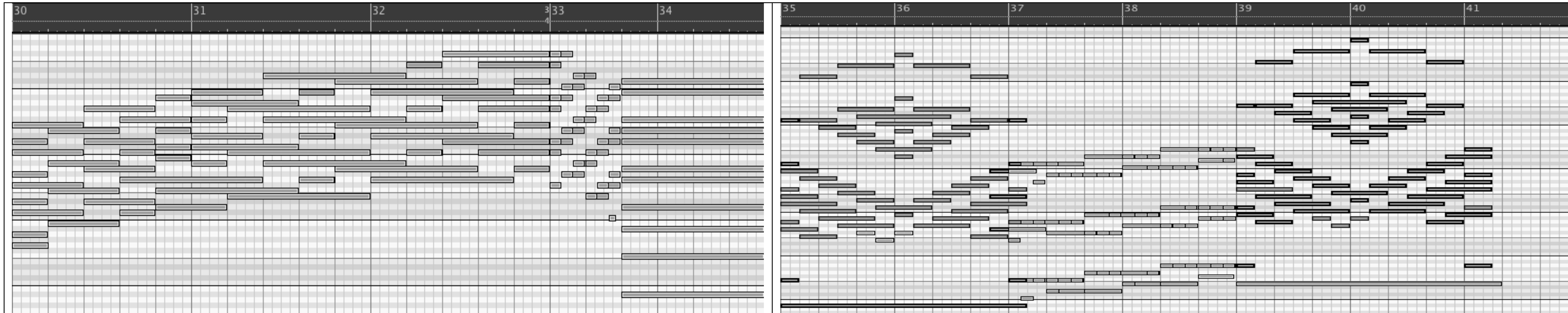


Form:	**Introduction**		**A**[1]			
Prime-Number:	**2**					
Cyclic Class:	16-5	32-7		16-2	32-7	16-2
Measure:	1-3	3-10	11-12	13-14	15-16	17-18

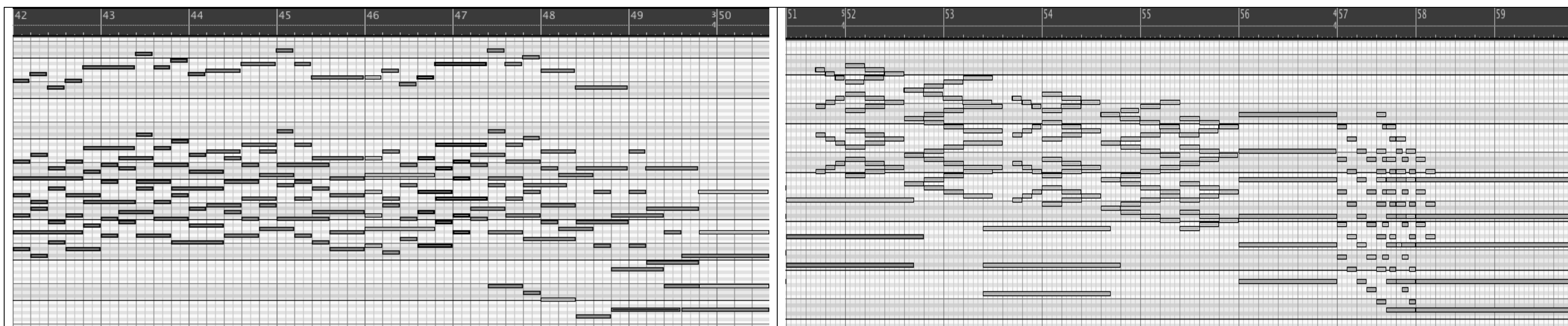


B		**C**	
11			
11-6	11-2	11-6	11-2
19-21	22	23-28	29

Figure A.2. Graphical and formal chart of Movement II.

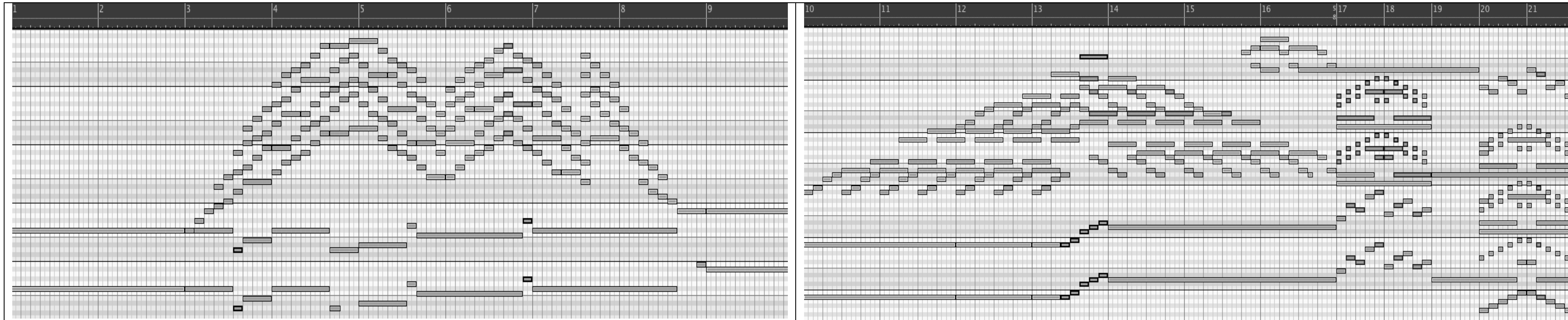


Form:	D			A^2		
Prime-Number:	11	2				
Cyclic Class:	11-4	8-3	16-5	32-7	16-2	32-7
Measure:	30	31-33	33-34	35-36	37-38	39-41

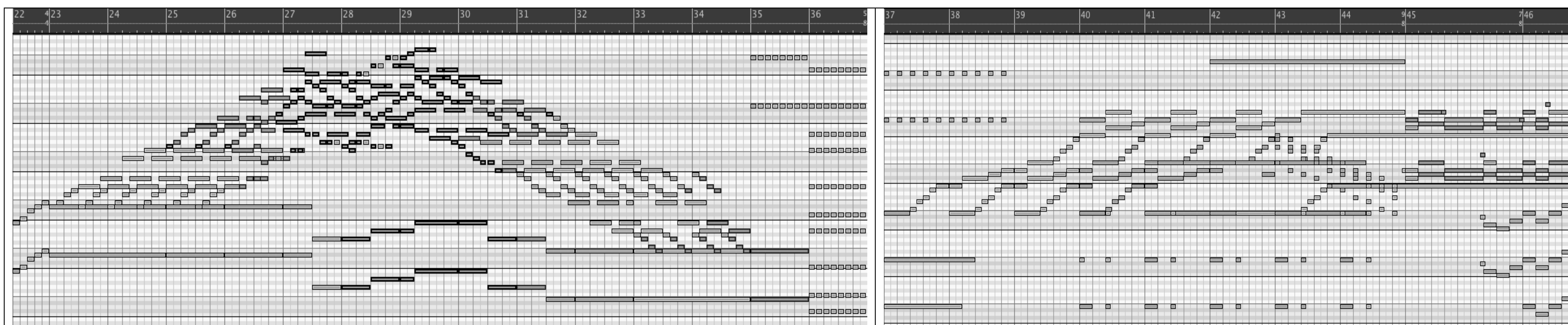


E		F
2		
16-3	16-2	16-2 and 16-4
42-48	48-50	51-59

Figure A.2. Continued.

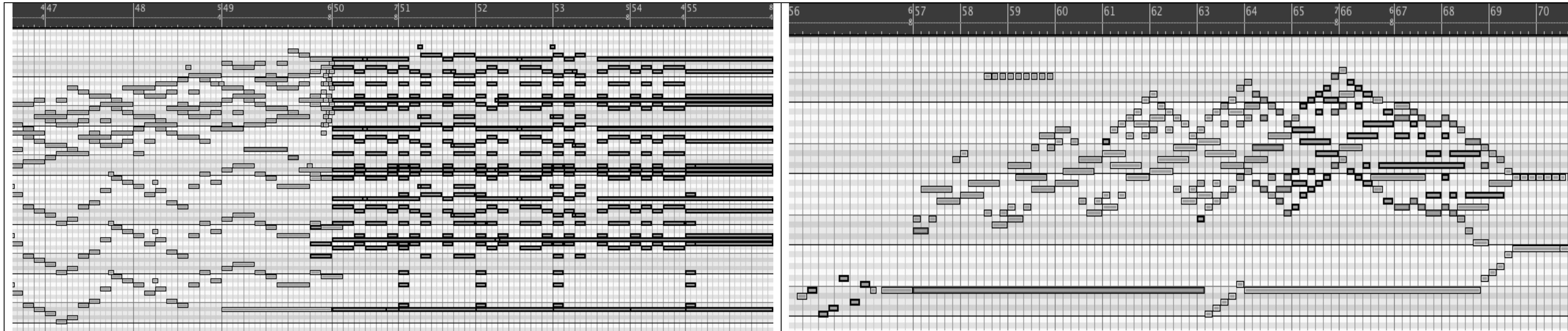


Form:	**A[1]**	**B[1]**	
Prime-Number:	3	5	13
Cyclic Class:	18-4	15-4	13-4
Measure:	1-9	10-16	17-22

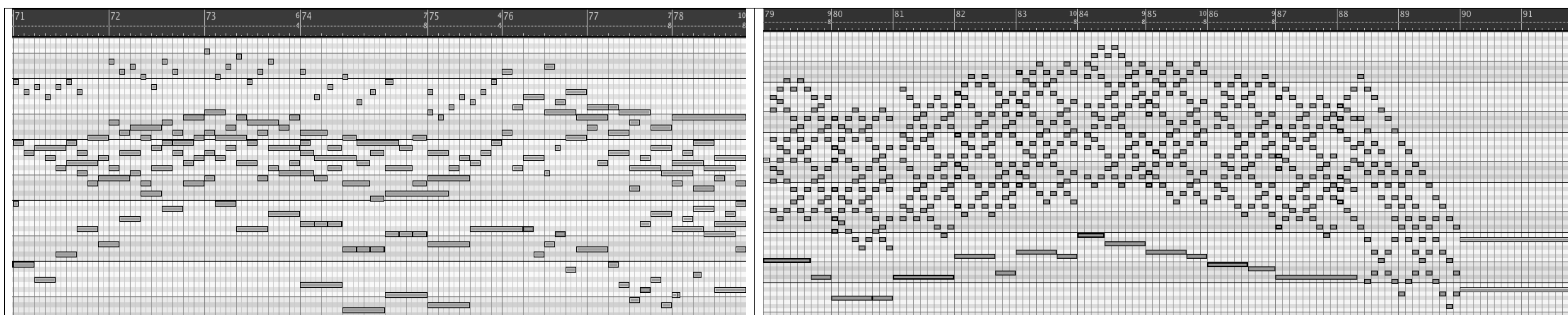


B[1] (continued)		**C[1]**	
5		3	13
15-4	20-3	24-4	13-3
23-35	36	37-44	45-46

Figure A.3. Graphical and formal chart of Movement III.

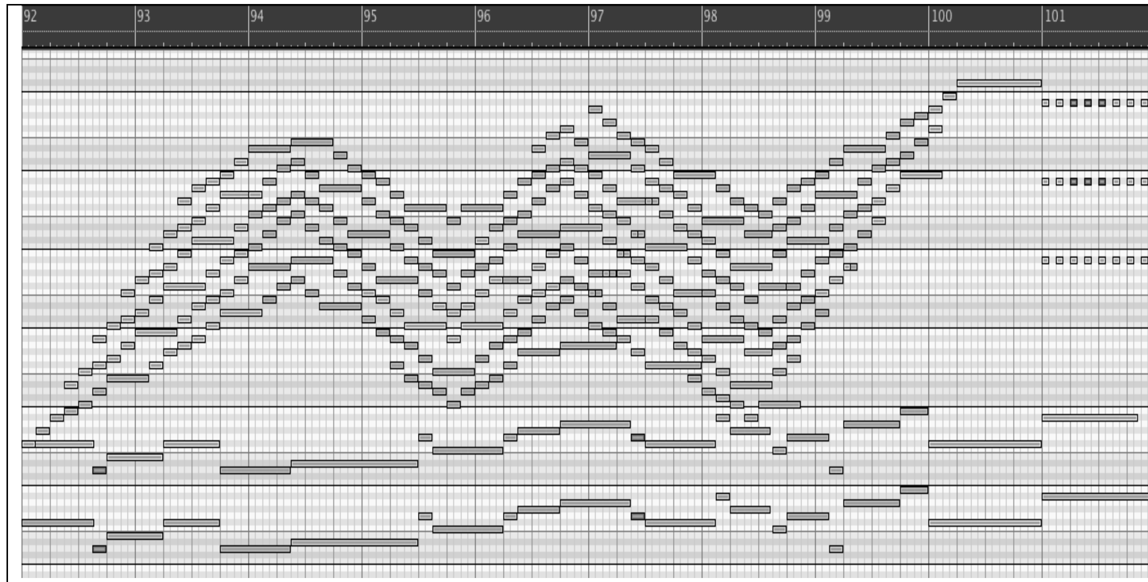


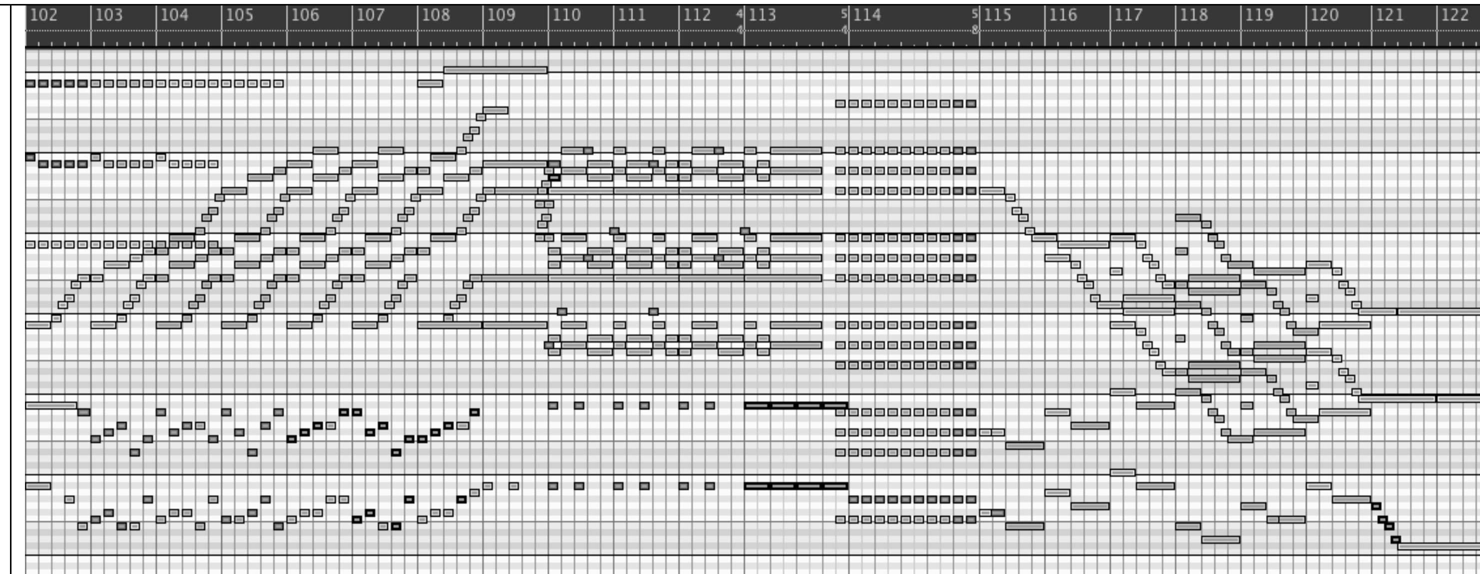
Form:	**D**[1]	**B**[2]
Prime-Number:	17	5
Cyclic Class:	17-5	15-4
Measure:	46-56	57-70



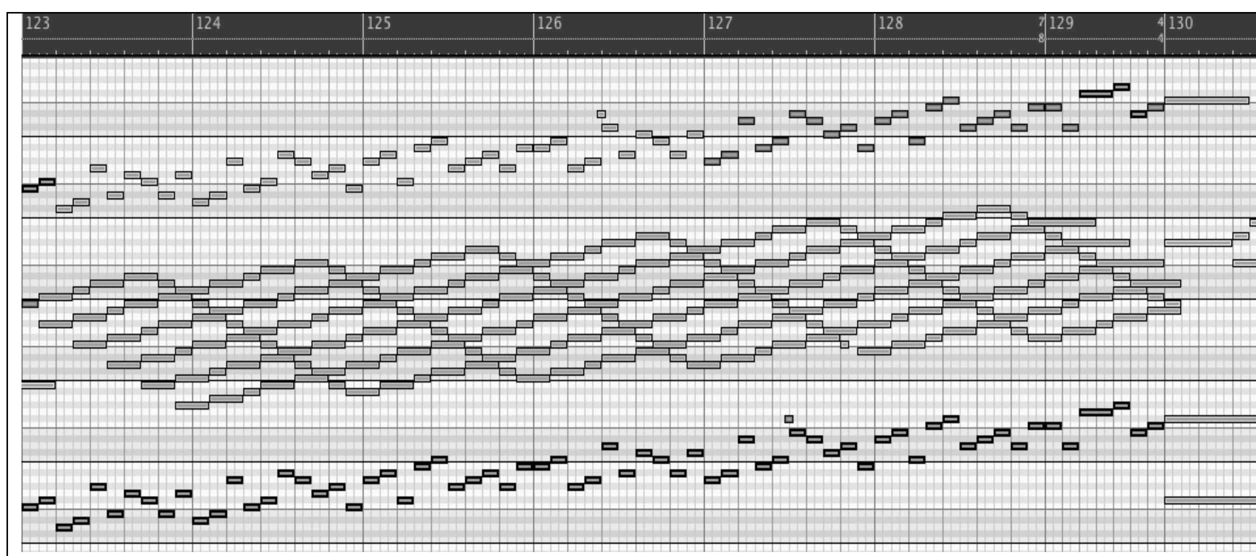
E	**F**	
13	17	13
13-3	17-3	13-4 and 13-5
71-78	79-88	88-91

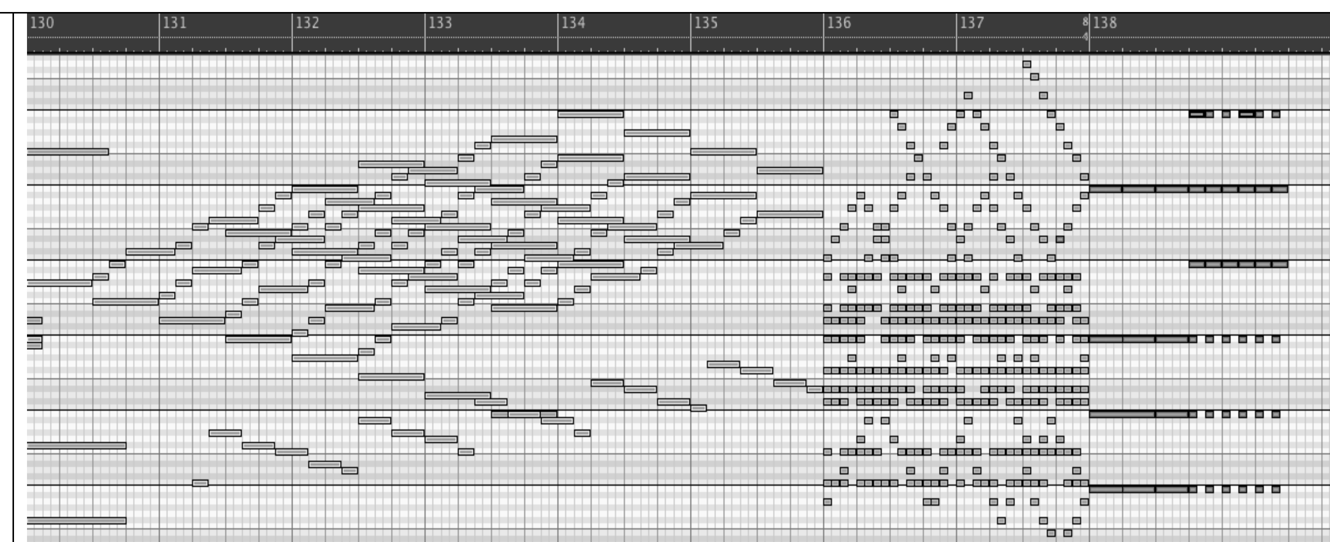
Figure A.3. Continued.





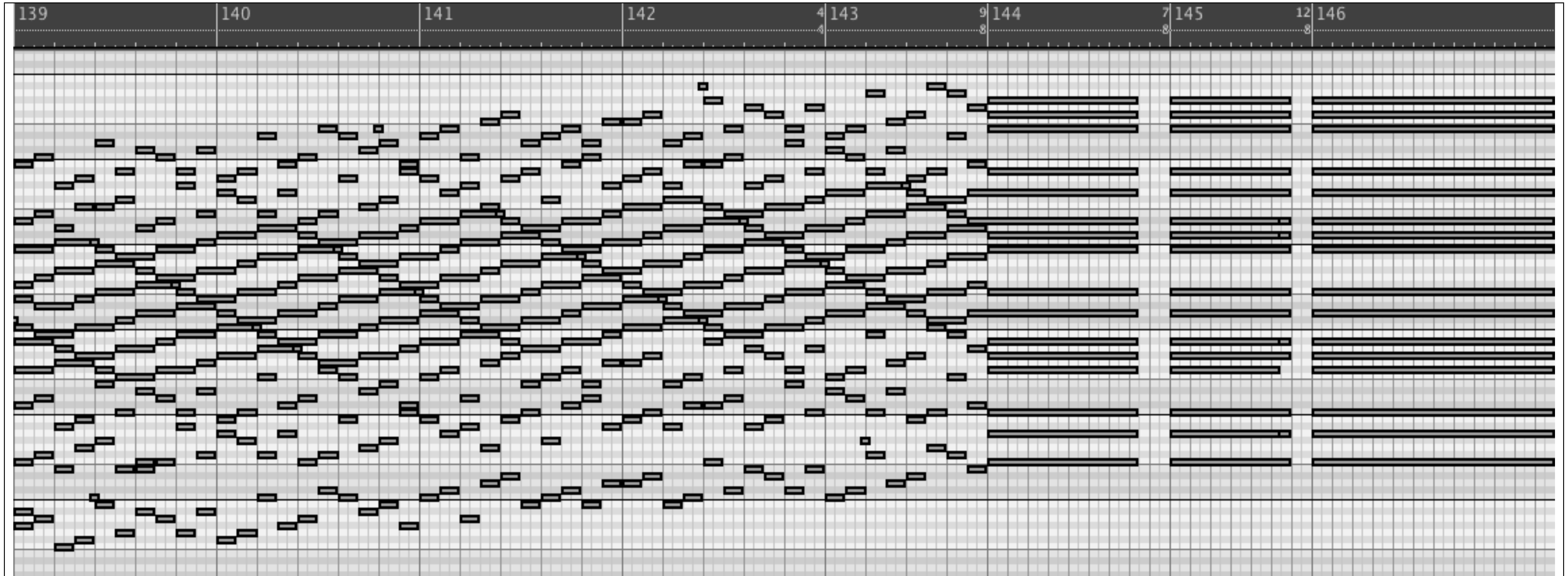
Form:	**A²**	**C²**			
Prime-Number:	3		13		
Cyclic Class:	18-4	24-4	13-2	13-3	13-1
Measure:	92-101	102-108	109	110-120	121-122





D²	
17	
17-5	
123-129	130-138

Figure A.3. Continued.



Form:	**D^2 (continued)**
Prime-Number:	17
Cyclic Class:	17-5
Measure:	139-146

Figure A.3. Continued.

APPENDIX B
MORE DISTANCES BETWEEN CYCLES

Table B.1. Some Distances between Cycles of Cardinality 2 going to Cycles of up to Cardinality 12.

Distance:	Cardinality Ratios:																
	2:2				**3:2**		**4:2**		**5:2**	**6:2**		**7:2**	**8:2**	**9:2**	**10:2**	**11:2**	**12:2**
	Partition Ratios:																
±0	2:2				3:2		4:2		5:2	6:2		7:2	8:2	9:2	10:2	11:2	12:2
±1	None				1:1 2:1 4:3 5:3 7:5 8:5 10:7 11:7 13:9 14:9	16:11 17:11 19:13 20:13 22:15 23:15 25:17 26:17 28:19 29:19	None		2:1 3:1 7:3 8:3 12:5 13:5 17:7 18:7 22:9 23:9	None		3:1 4:1 10:3 11:3 17:5 18:5 24:7 25:7 31:9 32:9	None	4:1 5:1 13:3 14:3 22:5 23:5 31:7 32:7 40:9 41:9	None	5:1 6:1 16:3 17:3 27:5 28:5 38:7 39:7 49:9 50:9	None
±2	2:1 1:2 3:2 2:3 4:3 3:4 5:4 4:5 6:5 5:6	7:6 6:7 8:7 7:8 9:8 8:9 10:9 9:10 11:10 10:11	12:11 11:12 13:12 12:13 14:13 13:14 15:14 14:15 16:15 15:16	17:16 16:17 18:17 17:18 19:18 18:19 20:19 19:20 21:20 20:21	2:2 4:2 5:4 7:4 8:6 10:6 11:8 13:8 14:10 16:10	17:12 19:12 20:14 22:14 23:16 25:16 26:18 28:18 29:20 31:20	1:1 3:1 3:2 5:2 5:3 7:3 7:4 9:4 9:5 11:5	11:6 13:6 13:7 15:7 15:8 17:8 17:9 19:9 19:10 21:10	4:2 6:2 9:4 11:4 14:6 16:6 19:8 21:8 24:10 26:10	2:1 4:1 5:2 7:2 8:3 10:3 11:4 13:4 14:5 16:5	17:6 19:6 20:7 22:7 23:8 25:8 26:9 28:9 29:10 31:10	6:2 8:2 13:4 15:4 20:6 22:6 27:8 29:8 34:10 36:10	3:1 5:1 7:2 9:2 11:3 13:3 15:4 17:4 19:5 21:54	8:2 10:2 17:4 19:4 26:6 28:6 35:8 37:8 44:10 46:10	4:1 6:1 9:2 11:2 14:3 16:3 19:4 21:4 24:5 26:5	10:2 12:2 21:4 23:4 32:6 34:6 43:8 45:8 54:10 56:10	5:1 7:1 11:2 13:2 17:3 19:3 23:4 25:4 29:5 31:5
±3	None				3:1 3:3 6:3 6:5 9:5 9:7 12:7 12:9 15:9 15:11	18:11 18:13 21:13 21:15 24:15 24:17 27:17 27:19 30:19 30:21	None		1:1 4:1 6:3 9:3 11:5 14:5 16:7 19:7 21:9 24:9	None		2:1 5:1 9:3 12:3 16:5 19:5 23:7 26:7 30:9 33:9	None	3:1 6:1 12:3 15:3 21:5 24:5 30:7 33:7 39:9 42:9	None	4:1 7:1 15:3 18:3 26:5 29:5 37:7 40:7 48:9 51:9	None

Table B.2. Some Distances between Cycles of Cardinality 3 going to Cycles of up to Cardinality 13.

Distance:	Cardinality Ratios:															
	3:3				**4:3**	**5:3**	**6:3**		**7:3**	**8:3**	**9:3**		**10:3**	**11:3**	**12:3**	**13:3**
	Partition Ratios:															
±0	3:3				4:3	5:3	6:3		7:3	8:3	9:3		10:3	11:3	12:3	13:3
±1	None				1:1 3:2 5:4 7:5 9:7 11:8 13:10 15:11 17:13 19:14	2:1 3:2 7:4 8:5 12:7 13:8 17:10 18:11 22:13 23:14	None		2:1 5:2 9:4 12:5 16:7 19:8 23:10 26:11 30:13 33:14	3:1 5:2 11:4 13:5 19:7 21:8 27:10 29:11 35:13 37:14	None		3:1 7:2 13:4 17:5 23:7 27:8 33:10 37:11 43:13 47:14	4:1 7:2 15:4 18:5 26:7 29:8 37:10 40:11 48:13 51:14	None	4:1 9:2 17:4 22:5 30:7 35:8 43:10 48:11 56:13 61:14
±2	None				2:1 2:2 6:4 6:5 10:7 10:8 14:10 14:11 18:13 18:14	1:1 4:2 6:4 9:5 11:7 14:8 16:10 19:11 21:13 24:14	None		3:1 4:2 10:4 11:5 17:7 18:8 24:10 25:11 31:13 32:14	2:1 6:2 10:4 14:5 18:7 22:8 26:10 30:11 34:13 38:14	None		4:1 6:2 14:4 16:5 24:7 26:8 34:10 36:11 44:13 46:14	3:1 8:2 14:4 19:5 25:7 30:8 36:10 41:11 47:13 52:14	None	5:1 8:2 18:4 21:5 31:7 34:8 44:10 47:11 57:13 60:14
±3	2:1 1:2 3:2 2:3 4:3 3:4 5:4 4:5 6:5 5:6	7:6 6:7 8:7 7:8 9:8 8:9 10:9 9:10 11:10 10:11	12:11 11:12 13:12 12:13 14:13 13:14 15:14 14:15 16:15 15:16	17:16 16:17 18:17 17:18 19:18 18:19 20:19 19:20 21:20 20:21	3:3 5:3 7:6 9:6 11:9 13:9 15:12 17:12 19:15 21:15	4:3 6:3 9:6 11:6 14:9 16:9 19:12 21:12 24:15 26:15	1:1 3:1 3:2 5:2 5:3 7:3 7:4 9:4 9:5 11:5	11:6 13:6 13:7 15:7 15:8 17:8 17:9 19:9 19:10 21:10	6:3 8:3 13:6 15:6 20:9 22:9 27:12 29:12 34:15 36:15	7:3 9:3 15:6 17:6 23:9 25:9 31:12 33:12 39:15 41:15	2:1 4:1 5:2 7:2 8:3 10:3 11:4 13:4 14:5 16:5	17:6 19:6 20:7 22:7 23:8 25:8 26:9 28:9 29:10 31:10	9:3 11:3 19:6 21:6 29:9 31:9 39:12 41:12 49:15 51:15	10:3 12:3 21:6 23:6 32:9 34:9 43:12 45:12 54:15 56:15	3:1 5:1 7:2 9:2 11:3 13:3 15:4 17:4 19:5 21:5	12:3 14:3 25:6 27:6 38:9 40:9 51:12 53:12 64:15 66:15

Table B.3. Some Distances between Cycles of Cardinality 4 going to Cycles of up to Cardinality 14.

	Cardinality Ratios:								
	5:4	**6:4**		**7:4**	**9:4**	**10:4**	**11:4**	**13:4**	**14:4**
Distance:	**Partition Ratios:**								
±0	5:4	6:4		7:4	9:4	10:4	11:4	13:4	14:4
±1	1:1	None		2:1	2:1	None	3:1	3:1	None
	4:3			5:3	7:3		8:3	10:3	
	6:5			9:5	11:5		14:5	16:5	
	9:7			12:7	16:7		19:7	23:7	
	11:9			16:9	20:9		25:9	29:9	
	14:11			19:11	25:11		30:11	36:11	
	16:13			23:13	29:13		36:13	42:13	
	19:15			26:15	34:15		41:15	49:15	
	21:17			30:17	38:17		47:17	55:17	
	24:19			33:19	43:19		52:19	62:19	
±2	2:2	1:1	16:11	3:2	4:2	2:1	5:2	6:2	3:1
	3:2	2:1	17:11	4:2	5:2	3:1	6:2	7:2	4:1
	7:6	4:3	19:13	10:6	13:6	7:3	16:6	19:6	10:3
	8:6	5:3	20:13	11:6	14:6	8:3	17:6	20:6	11:3
	12:10	7:5	22:15	17:10	22:10	12:5	27:10	32:10	17:5
	13:10	8:5	23:15	18:10	23:10	13:5	28:10	33:10	18:5
	17:14	10:7	25:17	24:14	31:14	17:7	38:14	45:14	24:7
	18:14	11:7	26:17	25:14	32:14	18:7	39:14	46:14	25:7
	22:18	13:9	28:19	31:18	40:18	22:9	49:18	58:18	31:9
	23:18	14:9	29:19	32:18	41:18	23:9	50:18	59:18	32:9
±3	2:1	None		1:1	3:1	None	2:1	4:1	None
	3:3			6:3	6:3		9:3	9:3	
	7:5			8:5	12:5		13:5	17:5	
	8:7			13:7	15:7		20:7	22:7	
	12:9			15:9	21:9		24:9	30:9	
	13:11			20:11	24:11		31:11	35:11	
	17:13			22:13	30:13		35:13	43:13	
	18:15			27:15	33:15		42:15	48:15	
	22:17			29:17	39:17		46:17	56:17	
	23:19			34:19	42:19		53:19	61:19	

Appendix C
Glossary

Cardinality: The number of intervals a cycle has per stacking. Thus, [1-1-2] has a cardinality of 3.

Cardinality Class: a class of interval cycles that share a common cardinality. For example, the tetrachord cyclic class 5-4 is equivalent to (~) 7-4 ~ 8-4 ~ 11-4, and so on regardless of partitions. The usefulness of this class is questionable.

Compound Melody: Two or more voices of a metacycle orchestrated for a single instrumental line.

Coprime: Two or more numbers that share no common factor (cannot be divided by any common number) besides one. For example, the only common factor of 2, 5, and 13 is one.

Cycle: See interval cycle.

Cycle Normal Form (CNF): The generalized designation given to an interval cycle that reflects stratum equivalence, rotational equivalence, and John Clough's notion of interval normal form (INF). Under stratum equivalence, interval cycles sharing the same interval pattern are considered equivalent regardless of how many stackings are present. For instance, the interval cycle [1-6] is equivalent to [1-6-1-6], [1-6-1-6-1-6], and so on. Under rotational equivalence, interval cycles sharing the same interval pattern are considered equivalent regardless of rotation so long as the relative positions of the intervals are identical. For instance, cycle [1-1-2] is equivalent to [1-2-1] and [2-1-1]. John Clough's Interval Normal Form (INF) roughly places the largest interval to the right and the smallest interval to the left. For instance [1-2-1] is depicted as [1-1-2] in INF.

Cyclically Adjacent Voices: polyphonic voices locked in parallel motion by an interval equal to the lcm of the partition of the cycles and/or cyclic classes used in a particular metacycle. For instance, the cyclically adjacent voices in a tonal progression would be in parallel octaves.

Cyclic Class (cc): A class of interval cycles that share a common partition *p* and cardinality *c* indicated by the designation *p-c*. For instance, [1-6], [2-5], and [3-4] are members of the cyclic class 7-2.

Cyclic Completion: A complete interval cycle is one in which the first pitch of the initial stacking is identical with the first pitch of the terminal stacking. The number of stackings a cycle must traverse in order to achieve completion is called its cyclic completion quotient. This quotient is determined by dividing the lcm of the partition p and 12 by p. (See also optimal interval cycle.)

$$\frac{\text{lcm of p and 12}}{\text{p}} = \text{cyclic completion quotient}$$

Cyclic Family: a family of cyclic classes whose partitions and cardinalities are multiples of an initial coprime cyclic class.

Cyclic Family Distance: The distance between two cycles measured by the number of voices different between them per LCM. Cyclic family distance is determined by the equation $ax - by = \pm d$ where the cyclic class *a-y* is adjacent to cyclic class *b-x* in a succession of vertically aligned interval cycles. This pair of cycles is related by a partition ratio of *a:b* and a cardinality ratio of *y:x* at a distance of *d*. In this equation, every value other than distance must be a positive integer.

Equivalence Relation: Often it is convenient to group together objects having some common property and consider them all in an identical way (for example, the fractions 4/2 and 2/1). Formally, it would be incorrect to simply 'declare them to be equal,' since equality alludes to something much more precise. Instead, the appropriate notion is that of an equivalence relation, whereby we can call such things equivalent. In equivalence relations, the symbol ~ means *is equivalent to*.

Factors: Integers that can divide (go into) another number. For instance, 1, 2, 3, 4, 6, and 12 are factors of 12 (See also prime factors.)

Integer: A number without a decimal.

Interval Cycle (or Cycle): An ordered pattern of intervals fixed in pitch space, which may be vertically or horizontally aligned. A particular interval cycle is one that is a member of a specific rotation class [1-6] ~ [6-1], and stratum class [1-6] ~ [1-6-1-6] ~ [1-6-1-6-1-6]. Theorists such as Philip Lambert and Richard Cohn prefer to use the term transposition cycle rather than interval cycle.

Interval Normal Form: See cycle normal form.

Interval Space: As opposed to interval class, registrally specific musical space in which intervals are measured by the actual number of semitones between them. Inversional and octave equivalency are not used in classifying interval cycles.

Line: A voice or voices orchestrated for an instrument. More than one voice can be orchestrated for a single line when that line is a compound melody or an arpeggio. A line may skip freely from one voice to another.

Mapping: when X stackings of one interval cycle contain the same number of voices as X stackings of another cycle per LCM.

Maximum Allowable Voice-Leading Distance: Every cycle is limited to a maximum potential voice-leading distance per LCM. To determine this distance, simply subtract the partition interval *p* per LCM by the cardinality *c* per LCM: $p - c = d$ where *d* is an absolute (positive and/or negative) measure of motion in semitones.

Metacycle: An ordered sequence of vertically aligned interval cycles and voice-leading paths. These interval cycles are usually members of the same cyclic family. The sequence imposes ordering upon both the individual interval cycles as well as the voice-leading paths from one to another. The sequence can be repeated, and at the same time voices can be added or subtracted from it to vary the texture. (See also metacyclic completion and optimal metacycle.)

Metacyclic Competition: occurs when the initial metacycle is transpositionally equivalent to and in the same rotation as the terminal metacycle. Both the order of the interval cycles and the order of the voice-leading paths are retained upon repetition of the sequence. Like interval cycles, metacycles can be rotated and are interlocked with successive repetitions vis-à-vis a common axis chord. (See also optimal metacycle.)

Non-Redundant Interval Cycle: A complete interval cycle without reiterated pitch-class content. (See also twelve-tone cycle.)

Optimal Interval Cycle: A cycle with a cyclic completion quotient of 12. In microtonal, equally-tempered scales, a cycle with a completion quotient equal to the pitch interval corresponding to the octave. In microtonal scales with no octave, all cycles. (See also cyclic completion.)

Optimal Metacycle: A metacycle that is complete after a number of repetitions (rotations) equal to the cardinality of the interval cycles per lcm.

Palindromic Interval Cycle: A cycle the identity of which is retained upon being retrograded. For example, cycle [2-5-2-2-5-2-14].

Partition: A partition of a number *n* is a way of writing *n* as a sum of other numbers. For example, the partitions of 4 are [1-1-1-1], [1-1-2], [1-3], [2-2], and [4]. In this case, 4 is the partition of all these cycles (they are members of partition class 4).

Partition Class: A class of interval cycles that share a common partition.

Pitch Interval: the actual distance between two pitches in pitch space irregardless of octave or inversional equivalency.

Prime Factors: The prime factors of a number are the prime numbers that when multiplied equal that number. For instance, the prime factors of 30 are (2 * 3 * 5), and the prime factors of 32 are (2 * 2 * 2 * 2 * 2).

Prime Number: a number whose only factors are one and itself. (See also factors.)

Prime-Number Class (or pnc): Normally, a class of partitions the highest prime factors of which are equivalent to a common prime number. For instance, the prime factors of 30 are (2 * 3 * 5). Since 5 is the highest of these, 30 is a member of pnc 5. A prime number class does not impose restrictions upon cardinalities. For instance the cyclic class 5-1 ~ 5-3 ~ 10-3 ~ 25-7, and so on. This is a very rough measure of similarity used only to organize large-scale forms such as movements. Less common is a **cardinality prime number class** whereby cyclic class 5-2 ~ 5-4 ~ 11-8 ~ 21-16, and so forth. (See also prime factors.)

Rotation: Rotation is analogous to the tonal concept of chordal inversion. For instance, C-E-G, G-C-E and A#-C#-F# are all "triads." When applied to a metacycle, the term refers to voices, whereby *rotation 0* replaces "root position," *rotation 1* replaces "first inversion," and so forth. The number rotations possible for any given metacycle is equivalent to the cardinality per LCM of the cycles within that metacycle.

Stacking: A stacking constitutes a single statement of an interval cycle pattern. For instance, one stacking of [2] would consist of pitches C and D a whole tone apart, while two stackings of [2] would be C, D, and E. Since adjacent stackings are interlocked, they share a common axis pitch, in the second example, the axis pitch is D.

Twelve-Tone Cycle: an interval cycle in which all twelve pitch-classes are stated *at least* once upon cyclic completion. A twelve-tone cycle may have pitch-class repetition upon competition, thus need not be non-redundant. Thus, a twelve-tone cycle is not to be confused with a twelv-tone row. A twelve-tone cycle need not necessarily be optimal, since it is certainly possible for twelve pitch classes to be stated within the span of fewer than 12 stackings. (See also cyclic completion.)

Two-Stacking Rule: At least two stackings of a cyclic pattern are sufficient for us to identify it with certainty.

Voice: A single horizontal line within a succession of metacycles.

Voice-Crossing Rule: When one voice X is moving towards another voice Y that is moving in the same direction or is sustained, the movement of X can be no greater than one less than the interval between X and Y. When two voices move in contrary motion towards each other, the sum of their absolute movements can be no greater than one less than the interval between them.

Voice-Leading Distance: The sum of the voice motion within a single LCM.

Voice-Leading Motion: When one vertically aligned interval cycle moves to another, the movement may be characterized by one or a combination of several different motions: common-tone (or sustained) motion, similar (or unidirectional) motion, contrary (or multidirectional) motion, and parallel (or equal) motion. However, motion exclusively by parallel is rarely used, since it results in transposition without rotation.

Voice-Leading Path: The specific voice-leading distance and motion (directions) taken from one distinct interval cycle to another within a metacycle. The second interval cycle is considered distinct even if transpositionally equivalent to the initial cycle, provided that it is rotated.

Wedge Voice Leading: A suspension of the voice-crossing rule that allows two voices to merge inward towards a unison, or to split outward from a unison, thus decreasing or increasing the cardinality of the cycles respectively.